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Watertown Arsenal Laboratory
 Report No. WAL 730/134
 Problem No. E-3.2

28 April 1944

GUN FORGING

Investigation of German 76.2 mm. A/T Gun (Pak 36) Converted from
 Captured Russian 76.2 Field Gun (Pk 36).

OBJECT

To investigate the design, metallurgical and bore characteristics
 of the subject gun tube.

SUMMARY OF RESULTS

1. The barrel consists of two sections - a rifled tube and a loose fitting outer jacket at the breech end which may contribute to the strength of the tube. The rifled tube is estimated to be capable of withstanding at the origin of rifling an internal pressure of 44,900 psi. (equivalent to a copper gage pressure of about 37,400 psi.) when completely unsupported by the jacket and a maximum of 54,600 psi. (equivalent to a copper gage pressure of about 45,600 psi.) if the clearance between the two members is zero. The outer jacket also serves as a breech locking collar by which the barrel is fastened to the breech ring.

2. For a given velocity the internal pressure is reported to be low [REDACTED], probably because of the particular interior ballistics. This feature permits the use of relatively low strength material which is generally associated with high resistance to impact. Although there are no data to indicate that this

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"The furtherance of a comprehensive shock and vibration program for the future needs of national defense depends upon the utilization of new ideas and novel techniques. Moreover, to keep our designers and technologists in the forefront of applied science, it is essential to disseminate pertinent information as soon as it becomes available."

The above words were written by Dr. Elias Klein in the Foreword to Shock and Vibration Bulletin Number 9 in April, 1948. At the present time we are preparing the 46th Shock and Vibration Bulletin, yet the mission stated by Dr. Klein more than 27 years ago has not changed. If anything, its significance has grown. The ninth Shock and Vibration Bulletin contained five technical papers. Some recent Bulletins have contained more than 20 times that number. The literature explosion in other technical journals is just as intense. This is one of the reasons that SVIC initiated this Digest in 1969 as a current awareness journal.

Hopefully, this Digest covers most sources of unclassified published literature. If we are inadvertently failing to scan any important sources, we would appreciate your telling us. In addition, we at SVIC are interested in learning of important work in progress that may result in advancements in technology. Contact us. Perhaps we can pay you a visit to learn details. We may even be able to provide information to assist in your research efforts.

H. C. P.

ACCESSION NO. 1.1

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VIBRATION AND NOISE CONTROL ENGINEERING CONFERENCE AND WORK SHOP - SIDNEY AUSTRALIA OCTOBER 11-12, 1976

A Conference and Workshop on Vibration and Noise Control Engineering will be held in Sydney, Australia on October 11 - 12, 1976. It will be organized as a National Conference of the Institution of Engineers, Australia - National Committee on Applied Mechanics and should be of interest and value to practicing engineers and architects.

Two types of paper will be offered. For conference sessions papers will concern new research, investigations and developments. For workshop sessions, shorter contributions will involve case histories, procedures, techniques, problems and their solutions. Presentations and the general organization of the program will emphasize applications for the practising engineer and architect.

All correspondence should be addressed to:
Vibration and Noise Control
Engineering,
The Institution of Engineers,
Australia
157 Gloucester Street,
Sydney, N. S. W. 2000.

ACOUSTICAL SOCIETY INITIATES STANDARDS PUBLICATION PROGRAM

At its meeting during the week of 4 November 1974 the Acoustical Society of America established a standards publication program. The production of standards by the Society as a public service is intended to provide the best technical guidelines for manufacturers, consumers, Federal, State and local Government, and the general public, and to provide for the timely distribution of standards. It is expected that most of the published standards will bear the approval of the American Nation-

al Standards Institute (ANSI). The standards are applicable to many broad areas: physical acoustics, mechanical shock and vibration, and bioacoustics, corresponding to the work of three ANSI Standards Committee S1, S2 and S3. The Acoustical Society is the sponsor of these ANSI Committees. (The American Society of Mechanical Engineers is co-sponsor of Standards Committees S2.)

The ASA Standards program will begin with the publication of a standard on hearing protectors (a revision of Z24.22 1957) to be designated as ASA STD 1-1975. (This standard has been designated S3.19 by the American National Standards Institute, which approved the standard on 14 August 1974). ASA STD 1-1975 specifies the psychophysical procedures, physical requirements and means of reporting results for measuring the protective and attenuation characteristics of wearable devices which are used to protect the auditory system against excessive noise.

The next standard to be published by the Acoustical Society will be ASA STD. 2-1975 Noise of Engine-Powered Equipment. This standard covers methods for measurement of the maximum noise emitted by equipment such as automobiles, buses, equipment used in residential areas, and powered recreational equipment. The third standard will be on Balance Quality of Rotating Rigid Bodies, which is the counterpart of the international standard on the same subject (ISO 1940).

ASA standards will be announced in the Journal of the Acoustical Society of America as well as in NOISE/NEWS and may be ordered as they become available. Suggestions gained in the use of these standards will be welcome and should be addressed to the Standards Secretariat, Acoustical Society of America, 335 East 45th Street, New York, New York 10017 ([212] 685-1940).

AVRIL BRENIG
Standards Manager
ASA Standards Secretariat

USE OF GALERKIN'S METHOD FOR VIBRATION PROBLEMS

H. LEIPHOLZ*

Galerkin's method is a technique for obtaining approximate solutions to differential equations for which initial value problems or boundary value problems have been posed. Due to the conditions of periodicity involved in vibrations, boundary value problems are always obtained; therefore, only boundary value problems dealing with vibrations will be considered in the following. These have the form

$$Ly(x) = f(x) \quad (1)$$

$$[Uy(x)]_{x=A}^{x=B} = \kappa \quad (2)$$

if they are inhomogeneous or

$$L[y(x), \lambda] = 0 \quad (3)$$

$$[U[y(x), \lambda]]_{x=A}^{x=B} = 0 \quad (4)$$

if they are homogeneous. Sometimes the problem is a mixed one, i.e., partly inhomogeneous, partly homogeneous.

In equations (1) - (4), L and U are differential operators, y is the dependent variable (vector), x is the independent variable (vector), f is a prescribed (vector) function, κ is a prescribed quantity (n - tuple), and λ is an eigenvalue (m - tuple) that appears in equation (3) but may or may not appear in the boundary conditions, equation (4); A and B mark the boundaries in the x -domain of the problem under investigation.

According to Galerkin's method, a complete set $\phi_k(x)$, $k = 1, 2, 3, \dots$; of coordinate functions (1) is chosen that satisfy the boundary conditions, equations (2) and (4). The

dependent variable y is developed into the series

$$y = \sum_k a_k \phi_k \quad (5)$$

which is supposed to converge uniformly. The coefficients a_k in equation (5) are elements of a tensor whose dimensions depend upon the dimensions of the vectors y and x . The basic assumption of Galerkin's method is that

$$Ly(x) - f(x) = 0 \quad (6)$$

is orthogonal to each individual coordinate function in the inhomogeneous case. Substitution of the value of y in equation (5) for y in equation (6) yields the following

$$\int_A^B \{L[\sum_k a_k \phi_k(x)] - f(x)\} \phi_\ell(x) dx = 0, \quad \ell = 1, 2, 3, \dots \quad (7)$$

The set of equations thus obtained is used to determine the unknown coefficients a_k . The equations may be linear or nonlinear, depending upon the nature of the operator L .

In the case of a homogeneous problem, equation (7) is replaced by

$$\int_A^B \{L[\sum_k a_k \phi_k(x, \lambda)]\} \phi_\ell(x, \lambda) dx = 0, \quad \ell = 1, 2, 3, \dots \quad (8)$$

This homogeneous set of equations is first solved for the eigenvalue λ . Only after λ is known can the coefficients a_k be evaluated up to a common, arbitrary factor.

Originally, equations (7) and (8) are infinite sets of equations; each equation of the sets has an infinite number of terms.

Galerkin's method assumes that the indices k and ℓ in equations (7) and (8) are restricted to the finite set of integers $1, 2, 3, \dots, n$. By virtue of this assumption, equations (7) and (8) are truncated to yield a finite number of

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equations with a finite number of terms. The truncated systems of equations can be solved to obtain approximate values of λ and/or approximate values of the coefficients for $k = 1, \dots, n$ and, therefore, an approximate solution of y in equation (5). How well or whether at all an approximate result obtained by Galerkin's method does converge toward the corresponding exact result if n is allowed to tend toward infinity must be clarified. One subject of theoretical studies concerning Galerkin's method, therefore, is the convergence problem.

Methods of approximation other than Galerkin's method are frequently used in connection with boundary value problems. The distinct features of the various methods and the degree to which one method might perhaps be in some way superior to others can be investigated by studying the relationship between Galerkin's and other methods.

The methods of Ritz and Galerkin have long been closely associated. When the two methods are applied to classical conservative engineering problems, they are essentially equivalent. In the following discussion, therefore, the method of Ritz will be considered as a variation of Galerkin's method. The advantage of using Galerkin's method is that it can be applied directly to the differential equation of the problem under investigation; the method of Ritz on the other hand requires that a certain functional with extremum properties related to the problem be found first. The great disadvantage of Galerkin's method in its classical form is that the space of admissible coordinate functions is much more restricted than that for the method of Ritz. In the case of Galerkin's method, the coordinate functions must satisfy all boundary conditions of the problem posed; the Ritz approach allows a wider class of coordinate functions for satisfying only the essential (geometric) boundary conditions. Since the disadvantage of the Galerkin's method is important in a practical sense, serious efforts have been made to modify or extend it, so that the same coordinate functions might be applied for both the method of Ritz and that of Galerkin. Hence, the theory of Galerkin's method includes an introduction to and justification of extended equations of Galerkin.

In the literature concerned with vibrations are many applications of the methods of Ritz

and Galerkin to the solution of specific problems. These applications also clearly reflect modifications of the two methods as a result of theoretical considerations. But there is little proof that vibration-oriented engineers have helped consolidate and extend the theoretical foundations of Galerkin's method. For example, in a number of papers Galerkin's method is used to solve nonlinear problems (2-9). The convergence problem has been considered in papers in which results obtained by the methods of Ritz and Galerkin have been compared with those obtained from experimental data (10-13). The relationship between Galerkin's method and other methods, especially the finite element method, has also been pointed out (14, 15), and both the possible extension of Galerkin's method (16-20) and the relaxation of boundary conditions (21, 22) have been considered. These papers by no means represent a complete list, but they do represent the use of Galerkin's method as a tool for solving specific vibration problems. The method may be applied in its most recent extension or its most sophisticated form.

Theoretical Foundations

Since Galerkin's method, which is also attributed to Bubnov, originated in Russia, it is not surprising that a great deal of theoretical research has been carried out there. A number of Russian authors could be mentioned, but Mikhlin is the predominant one (23, 24). Mikhlin has stressed that Galerkin's method is more general than the method of Ritz and that it is applicable to problems that are not based on an extremum principle. In such cases, the method of Ritz in its classical form would fail. Mikhlin has investigated the conditions of convergence of Galerkin's method for the case of non-selfadjoint (nonconservative) problems using functional analysis. A possible modification of the original method, so that boundary requirements for the coordinate functions can be relaxed, has been published (23). A detailed description of other approximate methods and their convergence problems is also given (23). No attempt has been made, however, to establish a relationship among the various methods.

Other papers have been published in Western Europe, the United States, and Canada as questions have been raised regarding vibrations of nonconservative systems, namely, first place systems subjected to follower forces. Such problems are encountered in structures exposed

to the impact of a fast-moving fluid or gas-- for example, air and spacecraft and pipelines-- and are therefore of great importance in modern technology.

Leipholz dealt extensively with convergence of Galerkin's method; he first applied the theory of infinite equations and matrices (25), and then the theory of integral equations and Green's functions (26). The convergence of Galerkin's method for quasi-linear boundary value problems has also been considered (27). Finally, a technique for convergence improvement has been studied (28).

The close relationship between Galerkin's method and an iterative approach to an integral equation was useful for studying related methods (29), and for developing an extended version of Galerkin's method. The extended method, which allows for a softening of boundary requirements for coordinate functions, thus contributes to the removal of the method's essential disadvantage (30, 31). Hybrid Galerkin equations (32), hybrid Ritz equations (33), and the convergence problem involved (34) have been used in recent attempts to study the idea of adjoint coordinating function systems. Surveys on Galerkin's methods have been published (35, 36). The most extensive treatment of Galerkin's method, specifically with respect to its relationship to other methods, has uncovered equivalences among the various methods (37).

Improvement of the ratio of convergence using functional analysis in the methods of Galerkin and Fitts has been studied by Nitsche (38), Schock (39), and Amann (40).

The relationship between the finite element method and Galerkin's method has also been considered thoroughly (41, 42, 43). This is of great practical importance because only when the finite element method is interpreted as a modified version of Galerkin's method can it be applied to the many mechanical systems for which the principle of conservation of energy does not hold.

In order to illustrate the preceding remarks and to make the trends in the development of Galerkin's method evident, the method will be interpreted as the mathematical formulation of the principle of virtual work. The following is based on Leipholz (37).

Consider an elastic body of surface S , boundary (edge) B , and subjected partly to conservative forces, partly to nonconservative follower forces, Q_x if they are distributed over the surface, P if they are distributed along the edges. Assume the body completely released from its supports. Then, the principle of virtual yields

$$\begin{aligned} & \int_T \left[\int_S \left(\frac{d}{dt} \left(\frac{\partial \mathcal{T}}{\partial \dot{w}_t} \right) + \frac{\delta U}{\delta w} - Q_x(w) \right) \delta w dS + \right. \\ & + \int_{B_S} [\mathcal{F}(w) \mathcal{D}(\delta w) - \mathcal{F}_1(w) \mathcal{D}_1(\delta w) \\ & - \mathcal{F}_1(\delta w) \mathcal{D}_1(w) - P(w) \delta w] dB + \\ & \left. + \int_{B_f} [\mathcal{F}(w) \mathcal{D}(\delta w) - P(w) \delta w] dB \right] dt = 0. \end{aligned} \quad (9)$$

In equation (9) t is time; T , the time interval of integration; \mathcal{T} the density of the kinetic energy; $\delta U / \delta w$, the functional derivation of the potential energy U (44); \mathcal{F} , the vector of all possible edge forces; \mathcal{D} , the vector of corresponding displacements; \mathcal{F}_1 , the vector of those edge forces set free by releasing the body from its support; and \mathcal{D}_1 , the vector of those edge displacements made possible by removing the supports. B_S are the supported edges of the body and B_f the free edges. Other quantities in equation (9) have already been identified.

Assume that

$$w = f(t) \gamma(x_1, x_2). \quad (10)$$

Then,

$$\left. \begin{aligned} Q_X(w) &= fQ_X(y), \quad P(w) = fP(y), \\ \frac{d}{dt} \left(\frac{\partial \mathcal{F}}{\partial w_t} \right) &= -\lambda f y, \\ \frac{\delta U}{\delta w} &= f \frac{\delta U}{\delta y}, \\ \delta w &= f \delta y, \end{aligned} \right\} \quad (11)$$

where λ is the square of the frequency of vibration. Equation (11) can be used to rewrite equation (9):

$$\begin{aligned} & \int_S \{-\lambda y + \frac{\delta U}{\delta y} + Q_X(y)\} \delta y dS + \int_{B_S} [\mathcal{F}(y) \\ & \mathcal{D}(\delta y) - \mathcal{F}_1(y) \mathcal{D}_1(\delta y) - \\ & - \mathcal{F}_1(\delta y) \mathcal{D}_1(y) - P(y) \delta y] dB + \int_{B_F} [\mathcal{F}(y) \\ & \mathcal{D}(\delta y) - P(y) \delta y] dB = 0. \end{aligned} \quad (12)$$

In order to obtain equation (12), the common factor $\int_T f^2 dt$ was suppressed.

In the most extended version of Galerkin's method

$$y = \sum_i a_i \phi_i, \quad \delta y = \sum_i \delta a_i \phi_i \quad (13)$$

in equation (12), assuming that the coordinate functions ϕ_i satisfy only in part the essential (geometric) boundary conditions and do not satisfy the natural (dynamic) boundary conditions. This procedure and the replacement of $\delta U / \delta y$ by the equivalent Euler operator $E(y)$ results in the extended equations of Galerkin:

$$\begin{aligned} & \int_S \{-\lambda (\sum_i a_i \phi_i) + E(\sum_i a_i \phi_i) + Q_X(\sum_i a_i \phi_i)\} \phi_j dS + \\ & + \int_{B_S} [\mathcal{F}(\sum_i a_i \phi_i) \mathcal{D}(\phi_j) - \mathcal{F}_1(\sum_i a_i \phi_i) \\ & \mathcal{D}_1(\phi_j) - \mathcal{F}_1(\phi_j) \mathcal{D}_1(\sum_i a_i \phi_i) - \\ & - P(\sum_i a_i \phi_i) \phi_j] dB + \int_{B_F} [\mathcal{F}(\sum_i a_i \phi_i) \mathcal{D}(\phi_j) \\ & - P(\sum_i a_i \phi_i) \phi_j] dB = 0, \\ & i, j = 1, 2, 3, \dots, n. \end{aligned} \quad (14)$$

It is again stressed that these equations are valid for coordinate functions that do not satisfy all boundary conditions. Hence, the disadvantage of Galerkin's method has been eliminated, since the extended Galerkin equations (14) are greatly freed from restrictions involved with the choice of coordinate functions.

If the coordinate functions ϕ_i were limited to satisfying all boundary conditions, every line integral in (14) would vanish, and the classical equations of Galerkin would again be obtained:

$$\begin{aligned} & \int_S \{-\lambda (\sum_i a_i \phi_i) + E(\sum_i a_i \phi_i) + Q_X(\sum_i a_i \phi_i)\} \phi_j dS = 0, \\ & i, j = 1, 2, 3, \dots, n. \end{aligned} \quad (15)$$

There is close relationship between Galerkin's method and that of Trefftz (37). Choose, for example, coordinate functions ϕ_i that satisfy

the differential equation

$$-\lambda y + E y + Q_x y = 0 \quad (16)$$

of the problem, but no boundary conditions; the surface integral in (14) would vanish, and the equations of Trefftz, involving only line integrals,

$$\begin{aligned} & \int_{B_s} [\mathcal{F}(\Sigma a_i \phi_i) \mathcal{D}(\phi_j) - \mathcal{F}_1(\Sigma a_i \phi_i) \\ & \mathcal{D}_1(\phi_j) - \mathcal{F}_1(\phi_j) \mathcal{D}_1(\Sigma a_i \phi_i) - \\ & P(\Sigma a_i \phi_i) \phi_j] dB + \int_{B_f} [\mathcal{F}(\Sigma a_i \phi_i) \mathcal{D}(\phi_j) \\ & - P(\Sigma a_i \phi_i) \phi_j] dB = 0 \end{aligned} \quad (17)$$

$$i, j = 1, 2, 3, \dots, n,$$

would remain.

Equations of mixed type are a link between the extended equations of Galerkin and the equations of Trefftz. These mixed equations can be obtained by choosing coordinate functions that satisfy in part differential equation (16) and in part the boundary conditions.

The equivalences between the methods of Ritz and Galerkin can be shown by integrating equation (12) by parts, i.e., using the integral theorem of Gauss:

$$\begin{aligned} \int_T \int_S \frac{\delta U}{\delta w} \delta y \, dS &= \int_T \int_S \delta \mathcal{U} \, dS dt - \int_B \mathcal{F}(y) \\ \mathcal{D}(\delta y) \, dB, \end{aligned} \quad (18)$$

in which \mathcal{U} is the density of the potential energy, and $B = B_s + B_f$. Using equation (18) in equation (12) and rearranging terms:

$$\begin{aligned} & \delta \left[\int_S \left(-\frac{\lambda}{2} y^2 + \mathcal{U} \right) dS - \int_B \mathcal{F}_1(y) \mathcal{D}_1(y) dB \right] \\ & + \int_S Q_x(y) \delta y \, dS - \int_B P(y) \delta y \, dB = 0. \end{aligned} \quad (19)$$

This relationship can be used to generate the extended equations of Ritz by applying equation (13), carrying out the variations prescribed in equation (19), and observing that

$$\delta \mathcal{U} = \sum_k D_k(y) D_k(\delta y), \quad (20)$$

where D_k are appropriate differential operators following from the specific nature of \mathcal{U} . The result is

$$\begin{aligned} & \int_S [-\lambda (\Sigma a_i \phi_i) \phi_j + \sum_k D_k(\Sigma a_i \phi_i) D_k(\phi_j)] dS - \\ & - \int_B [\mathcal{F}_1(\Sigma a_i \phi_i) \mathcal{D}_1(\phi_j) + \mathcal{F}_1(\phi_j) \\ & \mathcal{D}_1(\Sigma a_i \phi_i)] dB + \\ & + \int_S Q_x(\Sigma a_i \phi_i) \phi_j \, dS - \int_B P(\Sigma a_i \phi_i) \phi_j \, dB = 0, \end{aligned} \quad (21)$$

$$i, j = 1, 2, 3, \dots, n.$$

The extended equations of Ritz (21) are equivalent to the extended equations of Galerkin (14). In both cases, the problem may be nonconservative, and, in both cases, the coordinate functions ϕ_i may violate boundary conditions, especially the natural (dynamic) boundary conditions.

Assume that the problem is conservative. Then, $Q_x \equiv 0$, $P \equiv 0$; no nonconservative forces are present. Let the coordinate functions ϕ_i satisfy all essential (geometric) boundary conditions so that the line integral in equation (21) vanishes. As a result of these assumptions, the classical equations of Ritz

$$\int_S [-\lambda (\sum_i a_i \phi_i) \phi_j + \sum_k D_k (\sum_i a_i \phi_i) D_k (\phi_j)] dS = 0,$$

$$i, j = 1, 2, 3, \dots, n, \quad (22)$$

are obtained from equation (21).

These considerations show, that, in principle, there is no difference between the equations of Ritz and Galerkin. Regardless of the degree to which the boundary conditions are being satisfied or violated by the coordinate functions, and regardless of whether or not non-conservative forces are present, the two equations can be transformed into each other by proper integration procedures and by taking into account the necessary additional boundary terms in the equations. Hence, the use of equation (9) results in an extended and liberal interpretation of the method of Galerkin; use of this equation allows the relaxation of boundary conditions imposed on coordinate functions and the substitution of the Ritz, Galerkin, or Trefftz equations for each other.

Modification of the method of Galerkin by equation (9) reflects a trend in theoretical investigations.

Expansion Theorem

The application of Galerkin's method depends on the solution of y being expandable as

$$y = \sum_k a_k \phi_k. \quad (5)$$

In setting up Galerkin's equations, integration and summation operations are interchanged to ensure uniform convergence of (5). In that context, numerous expansion theorems have been derived (45, 46). Still another approach to the problem, in which reproducing kernels are used, is presented below.

Let $f(x)$ be a given function. Assume that this function satisfies

$$[Uf]_A^B = \kappa. \quad (23)$$

These conditions are considered as boundary conditions to a properly chosen, positive definite, selfadjoint differential operator L . Related to L is the eigenvalue problem

$$L\phi = \lambda\phi \quad (24)$$

$$[U\phi]_A^B = \kappa \quad (25)$$

yield the infinit set of eigenfunctions ϕ_i , $i = 1, 2, 3, \dots$. The ϕ_i satisfy equations (24) and (25), and λ_i is the eigenvalue corresponding to an individual eigenfunction ϕ_i . By virtue of its prescribed properties, operator L has the inverse

$$L^{-1}\psi(x) = \int_A^B K(x, \xi) \psi(\xi) d\xi, \quad (26)$$

whose kernel admits the absolutely convergent bilinear expansion

$$K(x, \xi) = \sum_i \frac{\phi_i(x) \phi_i(\xi)}{\lambda_i}. \quad (27)$$

Calculate

$$Lf(x) = h(x), \quad (28)$$

in which $h(x)$ is known. Premultiply both sides in equation (28) by L^{-1} and use equation (26) on the right hand side:

$$f(x) = \int_A^B K(x, \xi) h(\xi) d\xi. \quad (29)$$

The series expansion of $f(x)$ is obtained by substituting equation (27) in equation (29) and rearranging terms:

$$f(x) = \sum_i a_i \phi_i(x), \quad a_i = \int_A^B \frac{h(\xi) \phi_i(\xi)}{\lambda_i} d\xi. \quad (30)$$

Without any loss of generality, the set can be assumed to be orthonormal. Hence,

$$\int_A^B \phi_i(x) \phi_j(x) dx = \delta_{ij}, \quad (31)$$

in which δ_{ij} is the Kronecker symbol. Then,

$$a_i = \int_A^B f(x) \phi_i(x) dx. \quad (32)$$

This relationship must agree with equation (30). Therefore,

$$\int_A^B f(x) \phi_i(x) dx = \int_A^B \frac{h(\xi) \phi_i(\xi)}{\lambda_i} d\xi \quad (33)$$

must hold. Multiply both sides in equation (33) by λ_i , using equations (24) and (28), and change the integration variable ξ into x on the right hand side of (33) to obtain

$$\int_A^B f(x) L \phi_i(x) dx = \int_A^B \phi_i(x) L f(x) dx. \quad (34)$$

This relationship holds since the operator L is supposed to be selfadjoint.

Having shown that equation (30) is reasonable, it will now be shown that the expansion of equation (30) converges uniformly and absolutely. For this purpose, consider

$$|f(x) - \sum_{i=1}^N a_i \phi_i(x)|. \quad (35)$$

Substitute the right hand side of equation (29) for $f(x)$ in equation (35) and the expression in equation (30) for a_i and rearrange terms:

$$|f(x) - \sum_{i=1}^N a_i \phi_i(x)| = \left| \int_A^B [K(x, \xi) - \sum_{i=1}^N \frac{\phi_i(x) \phi_i(\xi)}{\lambda_i}] h(\xi) d\xi \right|. \quad (36)$$

therefore,

$$|f(x) - \sum_{i=1}^N a_i \phi_i(x)| \leq |K(x, \xi)| \sum_{i=1}^N \frac{|\phi_i(x) \phi_i(\xi)|}{\lambda_i} \cdot |h(\xi)|_{\max} \cdot C_1, \quad (37)$$

where C_1 is the "length" of the "x interval $[AB]$ ", i.e., a constant. Moreover, assume that $|h(\xi)|$ is bounded in $[AB]$ --for example, $|h(\xi)| < C_2$, in which C_2 is an appropriately chosen constant. Finally, take into account

$$|K(x, \xi) - \sum_{i=1}^N \frac{\phi_i(x) \phi_i(\xi)}{\lambda_i}| < \epsilon, \quad \epsilon > 0, \text{ small},$$

which is valid since K possesses a uniformly, absolutely convergent bilinear expansion by virtue of the properties of operator L .

Then, equation (37) can be rewritten as

$$\left. \begin{aligned} |f(x) - \sum_{i=1}^N a_i \phi_i(x)| &\leq C_1 \cdot C_2 \cdot \epsilon \text{ in } A \leq x \leq B, \\ \epsilon &\rightarrow 0 \text{ for } N \rightarrow \infty, \\ C_1, C_2 &\text{ constants,} \end{aligned} \right\} \quad (38)$$

which proves the uniform and absolute convergence of the series expansion, equation (30) in $A \leq x \leq B$.

The expansion theorem can be stated as follows:

Let a function $f(x)$ be given satisfying conditions in equation (23). Let a positive definite,

selfadjoint, differential operator L be chosen such that, by means of equations (24) and (25), the set of eigenfunctions ϕ_i and the reproducing kernel $K(x, \xi)$ are generated. $K(x, \xi)$ possesses the uniformly and absolutely convergent bilinear expansion, equation (27), by virtue of the properties of L . Let $h(x) = Lf(x)$ be bounded in $A \leq x \leq B$. Then, the series

$$f(x) = \sum_i a_i \phi_i(x), \quad a_i = \int_A^B \frac{h(\xi) \phi_i(\xi)}{\lambda_i} d\xi$$

is uniformly and absolutely convergent in $A \leq x \leq B$.

Incomplete Coordinate Systems

Take the case of equations (24) and (25) allow zero eigenvalues λ_0 . Then, the operator L is no longer positive definite (although it is still selfadjoint), and the homogeneous problem corresponding to equations (24) and (25) has nontrivial solutions ϕ_0 :

$$L\phi_0 = 0, \quad [U\phi_0]_A^B = \kappa. \quad (39)$$

The system of eigenfunctions ϕ_i corresponding to equations (24) and (25) becomes incomplete under these circumstances but can still be used to set up the expansion shown in equation (30). However, among the coefficients a_i of this expansion are also those for which

$$a_0 = \int_A^B \frac{h(\xi) \phi_0(\xi)}{\lambda_0} d\xi, \quad \lambda_0 \equiv 0 \quad (40)$$

holds. These coefficients correspond to the zero eigenvalues λ_0 of operator L and to the nontrivial solutions ϕ_0 of the homogeneous problem of equation (39). For a_0 to be meaningful:

$$\int_A^B h(\xi) \phi_0(\xi) d\xi \equiv 0. \quad (41)$$

Only if this is satisfied does a_0 in equation (40) remain a finite (but indeterminate) constant.

Hence the theorem:

Let a function $f(x)$ be given conditions satisfying equation (23). Let a selfadjoint differential operator L be chosen such that, by means of equations (24) and (25), the sets of eigenfunctions ϕ_i and eigenvalues λ_i are generated. Let the problem of equations (24) and (25) allow zero eigenvalues $\lambda_0 \equiv 0$ so that the homogeneous problem, equation (39), has nontrivial solutions ϕ_0 . Let $h(x) = Lf(x)$ be bounded in $A \leq x \leq B$ and be orthogonal to all nontrivial solutions ϕ_0 , thus satisfying equation (41). Then, and only then, the expansion

$$\left. \begin{aligned} f(x) &= \sum_{k=1}^m C_k \phi_{0,k} + \sum_{i=1}^{\infty} a_i \phi_i(x), \\ C_k &= \text{arbitrary constants}, \\ a_i &= \int_A^B \frac{h(\xi) \phi_i(\xi)}{\lambda_i} d\xi \end{aligned} \right\} \quad (42)$$

exists and is uniformly and absolutely convergent in $A \leq x \leq B$.

It may be noted that for expansion of $f(x)$ by means of the incomplete coordinate system ϕ_i , fulfillment of the condition in equation (41) is indispensable. Hence, no arbitrary function $f(x)$ would be expandable in the form of equation (42). Moreover, the infinite series $\sum_i a_i \phi_i$ involved in equation (42) must be complemented by the finite series $\sum_{k=1}^m C_k \phi_{0,k}$. The $\phi_{0,k}$, $k = 1, 2, \dots, m$, are the $m \geq 1$ nontrivial solutions of the problem expressed in (39). The uniform and absolute convergence of the infinite series $\sum_i a_i \phi_i$ can be proved as in the preceding section. For this purpose, the boundedness of h in $A \leq x \leq B$ is required.

As an example, consider the classical Fourier expansion of a periodic function $f(x)$ const.

which is in class C_2 . Hence,

$$f''(x) = h(x), \quad (43)$$

$$\left. \begin{aligned} f(0) &= f(2\pi), \\ f'(0) &= f'(2\pi). \end{aligned} \right\} \quad (44)$$

The operator L reads $L = d^2/dx^2$, and in place of equations (24) and (25) are:

$$\phi'' = \lambda \phi, \quad (45)$$

$$\left. \begin{aligned} \phi(0) &= \phi(2\pi), \\ \phi'(0) &= \phi'(2\pi). \end{aligned} \right\} \quad (46)$$

This boundary value problem generates the orthonormal eigenfunctions and eigenvalues

$$\phi_k = \frac{1}{\sqrt{\pi}} \sin kx, \quad \tilde{\phi}_k = \frac{1}{\sqrt{\pi}} \cos kx, \\ \lambda_k = -k^2,$$

$$k = 1, 2, 3, \dots \quad (47)$$

That the system of eigenfunctions is incomplete is apparent when the homogeneous problem

$$u'' = 0, \quad u(0) = u(2\pi), \quad u'(0) = u'(2\pi) \quad (48)$$

is considered.

It allows the nontrivial solution

$$u = a = \text{const.} \quad (49)$$

Hence, the condition in equation (41) becomes

$$\int_0^{2\pi} ah(x)dx = a \int_0^{2\pi} h(x)dx = a \int_0^{2\pi} f''(x)dx \quad (50)$$

By virtue of the second boundary condition expressed in equation (46), the condition shown in equation (50) is satisfied. The conclusion is that any arbitrary periodic function in C_2 is expandable by means of the incomplete system, equation (47).

Probably because equation (41) is automatically satisfied, it is usually overlooked. This is unfortunate because, for problems of a different and more general nature, the condition shown in equation (41) is decisive for the expandability of the prescribed function in terms of the incomplete system of coordinate functions under consideration.

The generating kernel of equations (45) and (46) is

$$K(x, \xi) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\sin kx \sin k\xi + \cos kx \cos k\xi}{-k^2}. \quad (51)$$

It is obtained by substituting equation (47) in equation (27). Hence, equations (29) and (43) yield

$$f(x) + \text{const} = \frac{1}{\pi} \int_0^{2\pi} \sum_{k=1}^{\infty} \frac{\sin kx \sin k\xi + \cos kx \cos k\xi}{-k^2} f''(\xi) d\xi, \quad (52)$$

Integrate the right side in (52) by parts and take boundary conditions and the identities

$$\int_0^{2\pi} f(x) dx = \int_0^{2\pi} f'(x) dx, \quad \int_0^{2\pi} f(x) (dx)^2 = \int_0^{2\pi} f'(x) dx = f(x)$$

into account to obtain

$$f(x) + \text{const} = \sum_{k=1}^{\infty} \left[\frac{\sin kx}{\pi} \int_0^{2\pi} \sin k\xi f(\xi) d\xi + \frac{\cos kx}{\pi} \int_0^{2\pi} \cos k\xi f(\xi) d\xi \right]. \quad (53)$$

Let

$$\begin{aligned} \frac{1}{\pi} \int_0^{2\pi} \cos k\xi f(\xi) d\xi &= a_k, \\ \frac{1}{\pi} \int_0^{2\pi} \sin k\xi f(\xi) d\xi &= b_k. \end{aligned} \quad (54)$$

Substituting equation (54) in equation (53):

$$f(x) + \text{const} = \sum_{k=1}^{\infty} [a_k \cos kx + b_k \sin kx]. \quad (55)$$

Integrate equation (55) over the interval

$$\begin{aligned} \int_0^{2\pi} f(x) dx + 2\pi \cdot \text{const} &= 0, \\ \text{const} &= -\frac{1}{2\pi} \int_0^{2\pi} f(x) dx = -a_0. \end{aligned} \quad (56)$$

Substitution of equation (56) in equation (55) yields

$$\begin{aligned} f(x) &= \frac{1}{2\pi} \int_0^{2\pi} f(x) dx + \sum_{k=1}^{\infty} [a_k \cos kx + b_k \sin kx], \\ f(x) &= a_0 + \sum_{k=1}^{\infty} [a_k \cos kx + b_k \sin kx]. \end{aligned} \quad (57)$$

This is the classical Fourier series of $f(x)$, which is in agreement with the first line in equation (42). The corresponding terms

are

$$a_0 \equiv C_1 \phi_{0,1}, \quad \phi_{0,1} = a = \text{const}, \quad C_1 = \text{const},$$

and

$$\sum_{k=1}^{\infty} [a_k \cos kx + b_k \sin kx] \equiv \sum_{i=1}^{\infty} a_i \phi_i(x).$$

Galerkin's Method and Harmonic Balance

The preceding considerations can be generalized to provide a sound basis for using an incomplete system of coordinate functions with Galerkin's method and the harmonic balance method.

Take the nonlinear (quasi-linear) boundary value problem

$$Ly(x) = h(y, y'), \quad (58)$$

$$[Uy(x)]_{x=A}^{x=B} = \kappa, \quad (59)$$

in which h is a nonlinear function of its arguments and L is selfadjoint operator. Let equations (24) and (25) yield the eigenfunctions ϕ_i and the eigenvalues λ_i . Let the homogeneous problem expressed in equation (39) yield the nontrivial solutions $\phi_{0,k}$. Then, according to equation (42),

$$y(x) = \sum_{k=1}^{\infty} C_k \phi_{0,k} + \sum_{i=1}^{\infty} a_i \phi_i(x) \quad (60)$$

if the infinite series $\sum_i [\phi_i(x) \phi_i(\xi)] / \lambda_i$ converges toward $K(x, \xi)$; i.e., if equation (27) holds, if $h(y, y')$ is bounded in $A \leq x \leq B$, and if

$$\int_A^B h(y, y') \phi_{0,k} d\xi = 0, \quad \text{for all } k. \quad (61)$$

From (42) follows also

$$a_i = \int_A \frac{h(y, y') \phi_i(\xi)}{\lambda_i} d\xi,$$

This can be rewritten:

$$a_i \lambda_i = \int_A h(y, y') \phi_i(\xi) d\xi.$$

Using Kronecker's symbol, the preceding equation can be converted into

$$\sum_j a_j \lambda_j \delta_{ij} = \int_A h(y, y') \phi_i(\xi) d\xi.$$

However,

$$\delta_{ij} = \int_A \phi_i \phi_j d\xi.$$

Therefore,

$$\sum_j a_j \int_A \lambda_j \phi_j \phi_i d\xi = \int_A h(y, y') \phi_i d\xi.$$

Use $\lambda_j \phi_j = L\phi_j$ to obtain

$$\sum_j a_j \int_A L\phi_j \phi_i d\xi = \int_A h(y, y') \phi_i d\xi.$$

Since L is a linear operator and since $L\phi_{0,k} = 0$:

$$\begin{aligned} \int_A L(\sum_k \phi_{0,k} + \sum_j a_j \phi_j) \phi_i d\xi &= \int_A h[(\sum_k \phi_{0,k} + \sum_j a_j \phi_j), \\ (\sum_k \phi_{0,k} + \sum_j a_j \phi_j)'] \phi_i d\xi. \end{aligned} \quad (62)$$

In order to obtain equation (62), y and y' on the right side were replaced by the series expansion shown in equation (60).

It is noteworthy that equation (62) would have been obtained by applying Galerkin's technique to equation (58) using the series in equation (60) for y and the system of coordinate functions ϕ_i for the orthogonality condition shown in equation (8). However, the condition expressed in equation (61) has to be taken into account. Therefore, in the special case under consideration, the complete system of Galerkin's equations is:

$$\begin{aligned} \int_A \{L(\sum_k \phi_{0,k} + \sum_j a_j \phi_j) - h[(\sum_k \phi_{0,k} + \sum_j a_j \phi_j), \\ (\sum_k \phi_{0,k} + \sum_j a_j \phi_j)']\} \phi_i d\xi = 0, \end{aligned}$$

$$\int_A \{h[(\sum_k \phi_{0,k} + \sum_j a_j \phi_j),$$

$$(\sum_k \phi_{0,k} + \sum_j a_j \phi_j)']\} \phi_{0,k} d\xi = 0,$$

$$k = 1, 2, \dots, m; \quad i, j = 1, 2, \dots, n.$$

(63)

The ideas developed above on the application of Galerkin's method using an incomplete system of coordinate functions are in response to a question raised (47) in connection with investigations concerning the vibrations of a missile subjected to a follower thrust acting at one of its free ends.

As an example of the application of equation (63), take Van der Pol's problem

$$h(y, y') = -y + (\alpha - \beta y^2)y',$$

$$y'' + y - (\alpha - \beta y^2)y' = 0,$$

$$y(0) = y(2\pi/\omega),$$

$$y'(0) = y'(2\pi/\omega).$$

The first two eigenfunctions of

$\phi'' = \lambda\phi$, $\phi(0) = \phi(2\pi/\omega)$, $\phi'(0) = \phi'(2\pi/\omega)$ are

$$\phi_1 = \sqrt{\frac{\omega}{\pi}} \sin \omega x, \quad \phi_2 = \sqrt{\frac{\omega}{\pi}} \cos \omega x,$$

and the nontrivial solution of the homogeneous problem

$$\phi''' = 0, \quad \phi(0) = \phi(2\pi/\omega), \quad \phi''(0) = \phi''(2\pi/\omega)$$

is the constant $\phi_0 = c$. Hence, equation (63) yields for $k = 1$, $i, j = 1, 2$ the equations of Galerkin

$$\begin{aligned} & \int_0^{2\pi/\omega} \{-a_1 \sqrt{\frac{\omega}{\pi}} \omega^2 \sin \omega x - a_2 \sqrt{\frac{\omega}{\pi}} \omega^2 \cos \omega x + c + a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x \\ & + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x - \alpha(a_1 \sqrt{\frac{\omega}{\pi}} \omega \cos \omega x - a_2 \sqrt{\frac{\omega}{\pi}} \omega \sin \omega x) \\ & + \beta(c + a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x)^2 (a_1 \sqrt{\frac{\omega}{\pi}} \omega \cos \omega x \\ & - a_2 \sqrt{\frac{\omega}{\pi}} \omega \sin \omega x)\} \sqrt{\frac{\omega}{\pi}} \sin \omega x dx = 0. \end{aligned}$$

$$\begin{aligned} & \int_0^{2\pi/\omega} \{-a_1 \sqrt{\frac{\omega}{\pi}} \omega^2 \sin \omega x - a_2 \sqrt{\frac{\omega}{\pi}} \omega^2 \cos \omega x + c + a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x \\ & + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x - \alpha(a_1 \sqrt{\frac{\omega}{\pi}} \omega \cos \omega x - a_2 \sqrt{\frac{\omega}{\pi}} \omega \sin \omega x) \\ & + \beta(c + a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x)^2 (a_1 \sqrt{\frac{\omega}{\pi}} \omega \cos \omega x \\ & - a_2 \sqrt{\frac{\omega}{\pi}} \omega \sin \omega x)\} \sqrt{\frac{\omega}{\pi}} \cos \omega x dx = 0, \end{aligned}$$

$$\begin{aligned} & \int_0^{2\pi/\omega} \{c + a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x - \alpha(a_1 \sqrt{\frac{\omega}{\pi}} \omega \cos \omega x \\ & - a_2 \sqrt{\frac{\omega}{\pi}} \omega \sin \omega x) + \beta(c + a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x \\ & + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x)^2 (a_1 \sqrt{\frac{\omega}{\pi}} \omega \cos \omega x - a_2 \sqrt{\frac{\omega}{\pi}} \omega \sin \omega x)\} dx = 0. \end{aligned}$$

After integration, the first two equations yield,

$$a_1(1 - \omega^2) + a_2[\alpha\omega - \frac{\beta\omega^2}{4\pi}(a_1^2 + a_2^2)] + \frac{c a_{1,2}}{2} = 0, \quad (64)$$

and the third equation results in

$$c[2a_1(1 + \frac{1}{4\pi}) + 2a_2(1 + \frac{1}{4\pi})] = 0.$$

Since $a_1 \neq 0$, $a_2 \neq 0$, the preceding equation enforces $c \equiv 0$. From equation (60) it follows that

$$y = a_1 \sqrt{\frac{\omega}{\pi}} \sin \omega x + a_2 \sqrt{\frac{\omega}{\pi}} \cos \omega x$$

or

$$y = A \cos(\omega x - \phi), \quad (65)$$

where

$$\tan \phi = \frac{a_1}{a_2}.$$

and

$$\mathcal{A}^2 = \frac{\omega}{\pi} (a_1^2 + a_2^2).$$

Use $c \equiv 0$ and the relationship $a_1^2 + a_2^2 = \pi \mathcal{A}^2 / \omega$ in equation (64) to obtain

$$a_1(1 - \omega^2) + a_2[\alpha\omega - \frac{\beta\omega}{4}\mathcal{A}^2] = 0.$$

If $a_1 \neq 0$, $a_2 \neq 0$, the preceding equation can be satisfied only if

$$\omega \equiv 1, \quad \mathcal{A} = 2\sqrt{\frac{\alpha}{\beta}} \quad (66)$$

Use equation (66) in equation (65) to obtain

$$y = 2\sqrt{\frac{\alpha}{\beta}} \cos(x - \phi) \quad (67)$$

as the solution of the Van der Pol problem in the first Galerkin approximation. This result has also been obtained elsewhere.

As has been shown, the applicability of Galerkin's method to

$$Ly = h(y, y') \quad (58)$$

depends on the boundedness of h and on L having an inverse L^{-1} that is bilinearly expandable in a convergent way. These properties of h and L ensure the expandability of y in a convergent fashion. If it can be shown that the methods of Galerkin and harmonic balance are equivalent insofar as they can be mutually transformed by simple algebraic operations (in the sense of functional analysis), then the conditions of convergence of the method of Galerkin are also sufficient to secure the convergence of the method of harmonic balance.

Assume in equation (58) that

$$L\phi = \lambda\phi \quad (24)$$

yields the eigenvector ϕ . Then, y in equation (58) allows, under certain conditions assumed to be satisfied, the series representation

$$y = \mathcal{A}\phi, \quad (68)$$

where \mathcal{A} is a coefficient vector, and the right side of equation (68) is the scalar product of the vectors \mathcal{A} and ϕ .

The "Fourier coefficient" of $h(y, y')$ is obtained, using equation (68), as

$$\int_A^B h(\mathcal{A}\phi, \mathcal{A}\phi')\phi dx = \Phi(\mathcal{A}), \quad (69)$$

where Φ is an appropriate function. Hence,

$$h = \Phi(\mathcal{A})\phi, \quad (70)$$

and equation (58) becomes

$$Ly = \Phi(\mathcal{A})\phi. \quad (71)$$

In addition,

$$\phi = \frac{1}{\mathcal{A}} y \quad (72)$$

this follows from equation (68). Use equation (72) in (71) to obtain

$$Ly = \frac{\Phi(\mathcal{A})}{\mathcal{A}} y. \quad (73)$$

In the method of harmonic balance this "linearized" differential equation is solved for y to obtain the approximate solution

$$y^* = \mathcal{A}\phi. \quad (74)$$

Apply the method of Galerkin to the problem. First use equation (68) in equation (58) to obtain

$$\mathcal{A}L\phi = h(\mathcal{A}\phi, \mathcal{A}\phi'). \quad (75)$$

Multiply both sides in equation (75) by ϕ and integrate the results according to equations (24) and (69) in

$$\mathcal{A}\lambda = \Phi(\mathcal{A}). \quad (76)$$

These are the Galerkin equations from which y is obtained; then the approximate solution in equation (74) is obtained.

In the final step the equivalence of the two methods is shown: the fundamental equation of the method of harmonic balance (73) is changed into

$$y = L^{-1} \left[\frac{\phi(\mathcal{A})}{\mathcal{A}} y \right]. \quad (77)$$

Substitute for L^{-1} equations (26) and (27); for y on the right side substitute equation (68) to change equation (77) into

$$y(x) = \frac{\phi(\mathcal{A})}{\mathcal{A}} \int_A^B \frac{\phi(x)\phi(\xi)}{\lambda} \mathcal{A} \phi(\xi) d\xi. \quad (78)$$

However, the ϕ are orthonormal. Therefore, equation (78) results in

$$y(x) = \frac{\phi(\mathcal{A})}{\lambda} \phi(x). \quad (79)$$

Hence,

$$y\lambda = \phi(\mathcal{A})\phi. \quad (80)$$

Use equation (74) in equation (80) to obtain

$$\lambda \mathcal{A} \phi = \phi(\mathcal{A})\phi.$$

This equation must hold for any ϕ . Therefore,

$$\mathcal{A}\lambda = \phi(\mathcal{A}),$$

which is equivalent to equation (76), the set of Galerkin's equations. The conclusion is that the two methods are variations of each other and yield the same results after the expandibility of y in terms of ϕ has been established.

Due to the equivalence of the two methods, the convergence problem can be solved conveniently for the method of Galerkin, and the convergence conditions thus obtained are also valid for the method of harmonic balance. This puts the method of harmonic balance, so frequently used to solve quasi-linear vibration problems, on a sound basis.

Acknowledgement

The support of this research by the National Research Council of Canada under Grant No. A7297 is gratefully acknowledged.

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XXXXXXXXXXXXXXXXXXXXXXXXXXXX SHORT COURSES

MARCH

APPLIED TIME SERIES ANALYSIS

Dates: Mar. 1-5, 1976

Place: Washington, D.C.

Objective: Preliminary concepts, data collection and processing, digital filtering, digital filter stability, statistical problems in data analysis, the fast Fourier transform, fast Fourier transform implementation, power spectral density calculations, ordinary coherence, partial coherence, multiple coherence, multiple input-single output transfer functions from data will be discussed. Special discussions regarding applications to structural analysis, acoustics, and ground shock will be included to suit special interests. Hardware and software considerations will be covered for both computer and minicomputer time-series analysis systems.

Contact: Mrs. Morello, Time/Data, 2855 Bowers Avenue, Santa Clara, CA 95051, (408-985-0700)

VIBRATIONS OF ENGINEERING STRUCTURES

Dates: Mar. 22-26, 1976

Place: Princeton, N.J.

Objective: Until recently only structures in earthquake zones were considered as subject to dynamic loads. In the last few years, dynamically sensitive structures have been built. Tall buildings, industrial chimneys, radio masts, nuclear power plants, and large marine structures such as off-shore platforms, are all structures for which the dynamic behavior must be investigated in design. This course on structural vibrations aims to give the design engineer a better understanding of the topic.

Principles involved and the various methods of approach to the problem will be emphasized rather than any detailed mathematical treatment. Modern methods of design, based upon the statistical approach, will also be discussed. Naturally some mathematical development will be included in the course, but this would not be beyond the graduate engineer with several years of design experience.

Contact: Mr. William W. Ellis, Director of Professional Education School of Engineering/Applied Science. Princeton University, Princeton, N.J. 08540 (609-452-4612)

GROUND TESTING AND SIMULATION

Dates: Mar. 1-5

Place: The University of Tennessee
Space Institute

Objective: The goal of this course is to provide current information concerning the role of ground environmental test facilities in aerospace systems development. Particular emphasis upon recent utilization of facilities for development of aerodynamic, propulsion, space and integrated systems, advanced test techniques and emerging test capabilities. Consideration of experience gained from testing and from not testing including what to expect from ground test, why ground before flight test and what ground test can and cannot do.

Contact: Technical: Dr. Robert L. Young, Associate Dean & Professor of Aerospace & Mechanical Engineering UTSI Ext. 211. Administrative: All administrative questions, including motel accommodations to Mr. Jules Bernard, Manager Short Course Program Ext. 276, The Univer. of Tenn. Space Inst. Tullhamo, TN 37388, (615-455-0631).

APRIL

LINEAR PROGRAMMING FOR ENGINEERS

Dates: Apr. 5-9

Place: The University of Texas at Austin

Objective: The objectives of this course are to familiarize participants with the characteristics of a linear programming problem so that they can recognize such problems in practice and be able to formulate them for solution, to show participants the usefulness of postoptimality sensitivity analyses and teach them how to perform such analyses, and finally, to instruct participants in the preparation of such problems for computer input, and to interpret computer solution output.

The emphasis in this course will be placed on problem solving under close supervision, including individual work on computer terminals which are remoted to the CDC 6600 computer. Participants will also solve problems using computer decks which they will be permitted to retain.

Contact: Engineering Institute of the College of Engineering, Ernest Hall 2, 102, The Univer. of Texas at Austin, Austin, TX 78712 (512-471-3506).

INTRODUCTION TO VIBRATION AND SHOCK TESTING, MEASUREMENT, ANALYSIS, AND CALIBRATION

Dates: Apr. 19-23

Place: Dayton, Ohio

Objective: The seminar will deal with seismic qualification of electrical equipment for safe shutdown of nuclear power plants, to IEEE 344-1975 and other specifications.

Vibration and shock aspects of weapons and various types of air, sea, and land vehicles will be covered along with vibration and shock in industrial operations.

Contact: Mr. Wayne Tustin, 22 E. Los Olivos, Santa Barbara, CA 93105, (805-963-1124)

JUNE

TORSIONAL VIBRATION OF MACHINES

Dates: June 15-16

Place: Chicago, Illinois

Objective: This seminar is devoted to the understanding and use of the technology to solve problems in torsional vibration of engines, drive lines, compressors and power transmitting and generating systems. Topics on the design, development and selection of equipments will be discussed along with case histories. Critical speed and response calculation and measurement techniques will be described. Demonstrations of experimental apparatus, instrumentation, and phenomena will be provided to enhance the understanding of the subject material

Contact: Dr. R. L. Eshleman, Director, The Vibration Institute, 5401 Katrine, Downers Grove, IL 60514, (312-654-2053).

DOCUMENT INFORMATION

Copies of articles abstracted are not available from the Shock and Vibration Information Center (except for those generated by SVIC) Inquiries should be directed to library resources, authors, or the original publishers. According to prefixed letters on document numbers, articles can be obtained from the following agencies:

- AD — Defense Documentation Center, Attn: DDC-TSR-1,
Reference Service Branch, Cameron Station,
Alexandria, Va. 22314 (Qualified users only)
- ASME — American Society of Mechanical Engineers,
345 E. 47th St., New York, N. Y. 10017
- NASA — National Aeronautics and Space Administration,
Scientific and Technical Information Division,
Washington, D. C. 20546
- NSA — Superintendent of Documents, U.S. Government Printing Office,
Washington, D. C. 20402 (or NTIS)
- PB
AD — National Technical Information Service, Dept. Commerce,
N Springfield, Va. 22151
- SAE — Society of Automotive Engineers,
2 Pennsylvania Plaza, New York, N. Y. 10001
- UM — University Microfilms
313 No. Fir St., Ann Arbor, Mich.

Patent descriptions should be requested from the U.S. Patent Office, Washington, D.C. 20231. Doctoral theses are available from University Microfilms.

Addresses following the authors' names in the abstracts refer only to the first author listed.

The list of Periodicals Scanned can be found in Issues 1 and 12 of Volumes 4, 5, 6 and 7.

ABSTRACTS FROM THE

ANALYSIS AND DESIGN

ANALOGS AND ANALOG COMPUTATION

(Also see No. 243)

ANALYTICAL METHODS

(Also see Nos. 191, 287, 327)

76-160

ACCELERATED CONVERGENCE FOR VIBRATION MODES USING THE SUBSTRUCTURE COUPLING METHOD AND FICTITIOUS COUPLING MASSES

Karpel, M. and Newman, M.

School of Engineering, Tel Aviv Univer.,
Ramat Aviv, Israel, Israel J. Tech., 13
(1-2) 55-62 (May 1975) 13 refs, 4 figs

Key Words: natural frequencies, mode
shapes, dynamic structural analysis

An improved analytical method for calculating the vibration modes of complex structures is presented in this paper. For a structural system divided into two subsystems, A and B, the vibration analysis is performed in two steps. First, a number of low frequency normal modes of substructure A, including its rigid body modes, are computed, while the degrees of freedom at connection to substructure B are free. These normal modes, coupled to a mathematical model of substructure B, are used to compute a number of lower frequency modes of the combined structure. The feature of this method is the usage of a suitable fictitious set of concentrated masses at the interconnection degrees of freedom which are positive for substructure A and negative for substructure B. The procedure for applying this technique to a structure composed of many substructures is discussed,

and numerical examples are presented showing rapid convergence which is relatively insensitive to the magnitude of the fictitious masses when they are above some minimum level.

76-161

SUBSTRUCTURE ENERGY METHOD FOR PREDICTION OF SPACE SHUTTLE MODAL DAMPING

Kana, D. D. and Unruh, J. F.

Southwest Research Institute, San Antonio,
Texas, J. Spacecraft and Rockets, 10 (5)
294-301 (May 1975) 6 figs, 4 refs

Key Words: modal damping, space stations,
energy methods

Results demonstrate the validity of a dissipative energy approach for predicting the damping of a four-component space shuttle model by modal parameters obtained from tests of individual components. A relationship between modal damping energy per cycle and peak strain (or kinetic) energy is determined empirically from test data for each component. Undamped analytical models of each component are developed and combined into a system model. Modal kinetic (or strain) energies are obtained from the model. These data and empirical damping curves are used to apportion the proper amount of damping energy to each component in a combined system mode. Some discrepancies in results occur because of incomplete modeling of connecting link mechanisms and anomalies in modal responses.

NUMERICAL ANALYSIS

CURRENT LITERATURE

76-162

NONLINEAR RELAY MODEL FOR POST-STALL OSCILLATIONS

Schoenstadt, A. L.

Naval Postgraduate School, Monterey, Calif.

J. Aircraft. 12 (7) 572-577 (July 1975) 9 figs, 1 ref

Key Words: oscillation, aircraft, vibration response, numerical analysis

Divergent oscillations in the post-stall region for a general class of aircraft were analyzed. A numerical investigation, on one aircraft, showed that a sufficiently large negative lift slope can lead to these oscillations. The post-stall behavior was modeled in a general class of aircraft.

OPTIMIZATION TECHNIQUES

(Also see No. 214)

STABILITY ANALYSIS

(Also see No. 265)

76-163

A GENERAL METHOD FOR STABILITY ANALYSIS OF ROTATING SHAFTS

Dimarogonas, A. D.

Ing.-Arch. 44 (1) 9-20, (March 1975) 7 figs, 20 refs

Key Words: stability methods, rotors, shafts

A general method is presented for the stability analysis of rotating shafts. A continuous rotor with any number of discontinuities and linear and nonlinear external forces has been modeled using a number of finite elements and degrees of freedom and a number of comparison functions chosen as the deformation energy and application of Lagrange's equations yielded the equations of motion. Due to the bearing

76-164

STABILITY OF ELASTIC RODS VIA LIAPUNOV'S SECOND METHOD

Leipholtz, H. H. E.

Ing.-Arch. 44 (1) 21-26 (March 1975) 3 figs, 6 refs

Key Words: stability methods, rods, internal damping, viscous damping

Stability of elastic rods subjected to nonconservative tangential follower forces is rigorously investigated by Liapunov's Second Method adapted to the specific nature of continuous systems. For such rods internal viscous damping should have a sufficiently large magnitude in order to ensure stability. This agrees with the fact that small damping destabilizes in the presence of nonconservative forces.

STATISTICAL METHODS

(Also see Nos. 206, 228)

VARIATIONAL METHODS

(Also see No. 219)

FINITE ELEMENT MODELING

(Also see Nos. 173, 218, 230, 255, 272, 273, 307, 314)

76-165

FREE VIBRATION ANALYSIS USING SUBSTRUCTURING

Gordon H. and Boresi, A. P.

ASCE J. Struc. Div. 101 (ST 12) 2627-2639 (Dec. 1975)

Key Words: free vibration, dynamic structural analysis, finite element techniques

The method of substructuring is applied to the solution of free vibration problems for large

structural systems. Modifications of substructure stiffness and mass matrices permit substructure mode shapes to conform more accurately to the complete system mode shapes and elastic and inertial effects of the surrounding structure on each substructure. Computational algorithms are used to determine the substructure mode shapes associated with the lowest natural frequencies. These algorithms are based on a iterative scheme having an automatic starting method. The synthesis method for determining system mode shapes from the substructure mode shapes is demonstrated. An example problem is solved and the solution compared to that obtained by a conventional method.

MODELING

(Also see Nos. 293, 294, 295, 318, 322, 323)

DIGITAL SIMULATION

(Also see No. 316)

DESIGN INFORMATION

(Also see Nos. 208, 257, 278, 321)

DESIGN TECHNIQUES

(Also see No. 209)

CRITERIA, STANDARDS AND SPECIFICATIONS

76-166

BACKGROUND DOCUMENT FOR INTERSTATE MOTOR CARRIER NOISE EMISSION REGULATIONS

Environmental Protection Agency, Washington D.C. Office of Noise Abatement and Control
EPA/550/9-74-017, 86 pp (Oct. 74)

Key Words: motor vehicle noise, noise reduction, regulations

This report presents factual information used by EPA which led to the requirements of the motor carrier regulation. Topics discussed are: regulatory strategy, technology for quieting in-use vehicles, costs, measurement methodology, and economic impact. Truck noise sources are analyzed to determine regulatory maximum levels

which are consistent with available retrofit technology. Data are presented on relative stringency between the engineering test used by industry and the roadside tests specified by EPA.

PB-242 554/4GA

76-167

INFORMATION ON FAA CERTIFICATION OF AIRCRAFT

Modig, C.

Informatics, Inc., Rockville, Md.,
EPA/550/9-75-022 84 pp (Jan. 1975)

Key Words: aircraft noise, regulations

This study provides an overview of FAA aircraft type certification regulations and the regulatory process through which aviation noise regulations are or could be implemented. Special reference is made to the regulatory process most relevant to transport category and/or turbine powered aircraft. The various types of certificates are covered. Tabular data for various types and models certified since 1969 are presented.

PB-242 583/3GA

Sponsor: EPA-68-01-3115

76-168

DESIGN STANDARDS FOR NOISE: A REVIEW OF THE BACKGROUND AND BASES OF MIL-STD-1474(MI)

Garinther, G.R., Hodge, D.C., Chaikin, G., and Rosenberg, D.M.

Human Engineering Lab Aberdeen Proving Ground Md., Rept. No. HEL-TM-12-75, 47 pp (March 1975)

Key Words: standards and codes, noise measurement, engine noise, aircraft noise

MIL-STD-1474(MI) (Mar. 1973) was the first design standard for noise in which all Army review activities concurred. The review treats the historical background of Army noise standards and the bases for the noise-limit provisions of MIL-STD-1474(MI); it also considers specified noise measurement procedures.
AD-A012 160/8GA

SURVEYS

(Also see No. 297)

76-169

EARTHQUAKE ENGINEERING: BUILDINGS, BRIDGES, DAMS, AND RELATED STRUCTURES VOLUME 1. 1964-Dec 1973 (A BIBLIOGRAPHY WITH ABSTRACTS 172)

Habercom, G. E.

National Technical Information Service,
Springfield, VA, Rept. for 1964-Dec 73, 177 pp
(Aug. 1975)

Key Words: bibliographies, earthquake damage, buildings, bridges, dams, nuclear power plants, earthquake resistant structures, seismic design

Seismic phenomena relative to buildings, bridges, dams, and other structures are investigated in these Government-sponsored research reports. Damage assessment is made and design inadequacies are revealed. Suggestions for structural improvements for seismic response are presented. Included are abstracts of reports on site selection and earthquake-proofing for atomic power plants.
NTIS/PS-75/632/0GA

See also NTIS/PS-75/633

76-170

EARTHQUAKE ENGINEERING: BUILDINGS, BRIDGES, DAMS, AND RELATED STRUCTURES VOLUME 2. 1974-Jul 1975 (A BIBLIOGRAPHY WITH ABSTRACTS 127)

Habercom, G. E.

National Technical Information Service,
Springfield, VA, Rept. for 1964-Jul 75,
132 pp (Aug. 1975)

Key Words: bibliographies, earthquake damage, buildings, bridges, dams, nuclear power plants, earthquake resistant structures, seismic design

Seismic phenomena relative to buildings, bridges, dams, and other structures are investigated in these Government-sponsored research reports. Damage assessment is made and design inadequacies are revealed. Suggestions for structural improvements for structural response are presented. Included are abstracts of

reports on site selection and earthquake-proofing for atomic power plants.

NTIS/PS-75/633/8GA

Supersedes COM-74-11141

See also NTIS/PS-75/633

76-171

FEDERAL MACHINERY NOISE RESEARCH, DEVELOPMENT, AND DEMONSTRATION: FY 73-FY 75

Final report

Environmental Protection Agency, Washington,
D. C. Office of Research and Development
EPA/600/2-75-008, 116 pp (May 1975)

Key Words: machinery noise, engine noise, gear noise, construction equipment, noise reduction

The Interagency Machinery Noise Research Panel has prepared this report summarizing the Federal government's machinery noise research, development, and demonstration activities. The Federal agencies which sponsor and/or conduct the major portion of these activities are represented on the panel. They are Department of Defense, National Bureau of Standards, National Science Foundation, Bureau of Mines, National Institute for occupational Safety and Health, and EPA. Department of Labor is also represented. Other agencies which sponsor machinery noise RD and D are the Department of Agriculture and Consumer Product Safety Commission. The report contains brief descriptions and fiscal data for the agencies' activities. Emphasis is on fiscal years 1973 through 1975. Also included are references and bibliographies of reports and publications which have resulted from the Federal machinery noise RD and D activities.
PB-243 523/8GA

76-172

AIRPORT NOISE (A BIBLIOGRAPHY WITH ABSTRACTS 138)

Habercom, G. E.

Rept. for 1964-Jun 75, 143 pp (June 1975)

Key Words: aircraft noise, airports, noise reduction, bibliographies

Aircraft created noise, noise intensity, noise exposure, and physiological effects, all in airport environments, are presented in these Government-sponsored research reports.
NTIS/PS-75/530/6GA

76-173**FINITE ELEMENTS IN STRUCTURAL ANALYSIS (A BIBLIOGRAPHY WITH ABSTRACTS 70)**

Grooms, D. W.

Rept. for 1972-Jun 75, 75 pp (June 1975)

Key Words: dynamic structural analysis, finite element technique, computer programs, bibliographies

Finite element analysis as applied to dynamic and static problems are analyzed as well as linear and nonlinear problems. Some computer programs for finite element analysis are also presented but for a more complete listing of software developed for structural problems see NTIS/PS-75/428, Structural Mechanics Software. NTIS/PS-75/496/0GA

76-174**FEDERAL AIRCRAFT NOISE RESEARCH, DEVELOPMENT, AND DEMONSTRATION PROGRAMS: FY 73-FY 75**

Final rept.

Environment Protection Agency, Washington, DC Office of Research and Development
EPA/600/2-75-003, 201 pp (March 1975)

Key Words: aircraft noise, noise reduction

This report prepared by The Interagency Aircraft Noise Research Panel was established by the Environmental Protection Agency to aid EPA in fulfilling its responsibility, provides an inventory of current and planned Federal aircraft noise RD&D programs. The report is organized by technical areas with each Federal agency's programs presented under the appropriate technical area. Emphasis is on fiscal years 1974 and 1975, but summary information on fiscal years 1973 and 1976 is also included. The Appendix contains detailed programmatic information as furnished by the Federal agencies on their aircraft related RD&D activities.
PB-241 904/2GA

76-175**MECHANICAL RESPONSE OF STRUCTURAL ELEMENTS TO DYNAMIC LOADS**

Interim scientific rept. 1 Apr 74-31 Mar 75
Herrmann, G.

Stanford Univer. Calif. Dept. of Applied Mechanics, AFOSR-TR-75-0814 12 pp (May 1975)

Key Words: composite materials, structural elements, dynamic response, shock waves, interaction: structure-fluid

This report summarized the research activities carried out under the subject grant during the period 1 April 1974 to 31 March 1975. Subjects discussed are fracture mechanics, composite materials, shock waves, fluid-solid interaction, and dynamic stability.
AD-A011 552/7GA
Sponsor: Grant AF-AFOSR-2669-74

76-176**FATIGUE BEHAVIOR OF TALL BUILDINGS, LITERATURE REVIEW**

Technical rept.

Yu, R., Ramachandra Rao, A., and Yao, J. T. P.
Purdue Univer., Lafayette, Ind. School of Civil Engineering, CE-STR-75-1, 39 pp (April 1975)

Key Words: reviews, buildings, wind-induced excitation, fatigue (materials)

A survey and study of the current literature related to the possible occurrence of wind-induced fatigue problems in modern tall buildings was undertaken. Topics covered in the report include wind load, random vibration of wind-excited building structures, fatigue of metals, fatigue behavior of welded connections, and random fatigue.
PB-242 797/9GA
Sponsor: NSF-GK-36196

76-177

**AUTOMOBILE IMPACT TESTS (A
BIBLIOGRAPHY WITH ABSTRACTS)**

Rept. for 1964-Aug 1975

Adams, G. H.

National Technical Information Service,
Springfield, VA, 191 pp (August 1975)

Key Words: bibliographies, collision
research (automotive), crashworthiness,
anthropomorphic dummies, mathematical
models, test models

The bibliography covers tests results on the crashworthiness and safety of passenger vehicles upon impact with stationary structures, other vehicles, median berms, and curbs. Responses of vehicle structural components are studied in displacement and strain measurements and in detailed examinations of permanently deformed components following individual tests. Research involves vehicle impacts front to front, lateral, at angles, and onside; and collisions involving poles, bridge parapets, highway structures, and safety and test barriers. Anthropomorphic tests are described using experimental test vehicles, rocket-propelled sleds, and safety harness. Computerized simulation, mathematical models, and scale model studies are described. NTIS/PS-75/651/0GA

76-178

**FEDERAL SURFACE VEHICLE NOISE
RESEARCH, DEVELOPMENT, AND
DEMONSTRATION PROGRAMS: FY 73-FY 75**
Final report

Environmental Protection Agency, Washington
DC, Office of Research and Development
EPA/600/2-75-002, 96 pp (March 1975)

Key Words: ground vehicles, noise generation

The Interagency Surface Vehicle Noise Research Panel was established by the Environmental Protection Agency to aid EPA in fulfilling its responsibility for coordinating the Federal noise research activities. As its initial task, the Panel prepared this report summarizing the Federal government's surface vehicle noise research, development, and demonstration activities. The report contains brief descriptions and fiscal data for Federal agencies' programs. Emphasis is on fiscal

years 1973 through 1975. Also included are references and bibliographies of reports and publications which have resulted from the Federal surface vehicle RD&D activities. PB-241 887/9GA

COMPUTER PROGRAMS

GENERAL

(Also see Nos. 211, 230, 301, 313, 325, 328)

76-179

**A COMPUTER PROGRAM FOR PREDICTING
AIRLOADS ON A SINGLE OSCILLATING ROTOR
BLADE IN HOVER**

Schatzie, P. R. and Rao, B. M.

Texas Engineering Experiment Station College
Station, Rept. no. TEES-3099-75-01
ARO-11695.1-E, 23 pp (March 1975)

Key Words: computer programs, rotory wings,
aerodynamic loads

A FORTRAN computer program is described which predicts the unsteady airloads on a single oscillating rotor blade in hover using the numerical lifting surface technique developed by Rao and Jones. Typical results for the aerodynamic derivatives for a range of Mach number, wake spacing, and frequency ratio are given. The aerodynamic derivatives are calculated with respect to the mid-chord position and may be referred to any other reference axis by a transformation also given in the report. AD-A009 791/5GA

76-180

**BOEING AIRPLANE/NOISE PERFORMANCE
COMPUTER PROGRAM. USER'S MANUAL**
Final report

Bhatia, K., Jaeger, M. A., Williams, B. G.,
and Yates, R.

Boeing Computer Services Inc., Seattle, Wash.
Rept. no. BCS-G0422 FAA-EQ-73-7-6, 61 pp
(Dec. 1973)

Key Words: aircraft noise, noise reduction,
computer programs

Program usage for the Boeing Airplane Noise/Performance Computer Program is described. The program calculates takeoff and approach profiles, including noise-abatement procedures; output data are given for distance, height, speed, thrust, rate of climb, gradient, deck angle, engine pressure ratio, rotor speed, and noise under the flightpath in units of EPNdb and dB(A). A sample run of the program illustrates input requirements and output capabilities.

AD-A012 385/1GA

76-181

STATIC AND DYNAMIC ANALYSIS OF NONLINEAR STRUCTURES

Mondkar, D. P. and Powell, G. H.
California Univer. Berkeley, Earthquake Engineering Research Center, EERC-75-10, 168 pp (June 11, 1975)

Key Words: dynamic structural analysis, computer programs, nonlinear structures

Theoretical and computational procedures are given that have been applied in designing a general purpose nonlinear computer code for static and dynamic analysis of nonlinear structures. The principle of virtual displacements is used to derive incremental equations of placements and strains; the Lagrangian description of deformation is employed. The equations of motion are discretized using the finite element displacement formulation; the characteristics, including 'linear' and 'non-linear' stiffness matrices, are derived for a two-dimensional isoparametric finite element. Physical nonlinearity due to material behavior is introduced for elasto-plastic materials; the constitutive laws for commonly used elasto-plastic models are described.

PB-242 434/9GA

76-182

COMPUTER PROGRAM FOR CURVED BRIDGES ON FLEXIBLE BENTS

Final report

Kabir, A. F. and Scordelis, A. C.
California Univer., Berkeley, Div. of Structural Engineering and Structural Mechanics, UCSESM-74-10, 173 pp, (September 1974)

Key Words: computer programs, bridges, plates, stiffness methods, harmonic analysis, computer programs

A computer program is presented for the analysis of continuous prismatic folded plate structures, which are circular in plan and may have up to twelve flexible interior diaphragms or supports. The folded plate structure is considered to be an assemblage of orthotropic plate elements that may, in general, be segments of conical frustra, interconnected at longitudinal joints and simply supported at the two ends. Each plate element is idealized by a number of circumferential finite strips. The finite strip method is used to determine the strip stiffness, interior diaphragms may be defined by flexible beams, and interior supports may be defined by two-dimensional planar frame bents. A direct stiffness harmonic analysis is used to analyze the assembled folded plate system.

PB-242 470/3GA

Sponsor: Prepared in cooperation with California State Dept. of Transportation, Sacramento

76-183

COMMERCIAL AIRCRAFT NOISE DEFINITION L1011 TRISTAR. VOLUME 1.

Final Report

Shapiro, N.

Lockhead-California Co. Burbank, LR-26075-Vol-1 FAA-EQ-73-6-Vol-1, 85 pp, (Sept. 1975)

Key Words: commercial aircraft, aircraft noise, acoustic signatures, computer programs

Calculation procedures to describe airplane noise during takeoff and approach have been programmed for batch operation on a large digital computer. Three routines are included. The first normalizes far-field noise spectra to reference conditions and then determines spectra at various distances from the airplane, for airport elevations between sea level and 6000 feet and ambient temperatures between 30F and 100F. Overall sound pressure levels, A-weighted noise levels, perceived noise levels, and effective perceived noise levels are calculated. The second routine uses aerodynamic and engine thrust data to produce takeoff and approach flight path description. The basic takeoff is at equivalent airspeed, with thrust reduction or acceleration option after gear-up. The approach is along any constant glide slope between 3 and 6 degrees at constant airspeed with a two-segment option. The last routine combines noise propagation and flight path information to produce constant noise contour footprints. The program has been exercised on Lockheed L-1011-1 Tristar/Rolls-Royce RD. 211-22 data, providing results in EPNdB and dBA.

AD-A012 371/1GA

Sponsor: Contract DOT-FA73WA-3300

76-184

COMMERICAL AIRCRAFT NOISE DEFINITION
L-1011 TRISTAR. VOLUME II-L-1011-1 DATA
Shapiro, N

Lockheed-California Co., Burbank

Rept. no. LR-26075-Vol-2 FAA-EQ-73-6-
Vol-2, 302 pp, (Sept. 1974)

Key Words: commercial aircraft, aircraft
noise, acoustic signatures, computer programs

Volume 2 includes L-1011-1 noise propagation
and airplane performance and samples of
contours. (See also 76-183)

AD-A012 372/9GA

Sponsor: Contract DOT-FA73WA-3300

76-185

COMMERICAL AIRCRAFT NOISE DEFINITION.
L-1011 TRISTAR. VOLUME III - PROGRAM
USER'S MANUAL

Final report

Rept. no. LR-26075-Vol-3 FAA-EQ-73-6-
Vol-3, 105 pp, (Sept. 1974)

Key Words: commercial aircraft, aircraft
noise, computer programs, acoustic
signatures

Volume 3 presents the logic behind the calcu-
lations and outlines the computational proce-
dures. (See also 76-183)

AD-A012 373/7GA

76-186

COMMERCIAL AIRCRAFT NOISE DEFINITION
L-1011 TRISTAR. VOLUME IV-PROGRAM
DESIGN SPECIFICATION

Final report

Rept. no. LR-26075-Vol-4 FAA-EQ-73-6-
Vol-4, 126 pp, (Sept. 1974)

Key Words: commercial aircraft, aircraft
noise, acoustic signatures, computer programs

Volume 4 describes the computer program and
gives instructions for its operation. (See also
76-183)

AD-A012 374/5GA

76-187

COMMERCIAL AIRCRAFT NOISE DEFINITION
L-1011 TRISTAR. VOLUME V-COMPUTER
PROGRAMMER'S MANUAL

Final report

Rept. no. LR-26075-Vol-5 FAA-EQ-73-6-
Vol-5, 370 pp, (Sept. 1974)

Key Words: commercial aircraft, aircraft
noise, acoustic signatures, computer programs

Volume 5 describes the computer program and
gives instructions for its operation. (See also
76-183)

AD-A012 375/2GA

ENVIRONMENTS

ACOUSTIC

Also see Nos. 166, 167, 168, 171, 172, 174, 180, 183, 184, 185, 186, 187, 235, 260, 264, 281, 292, 296, 302)

76-188

ISOLATED AIRFOIL-TIP VORTEX INTERACTION NOISE

Paterson, R.W., Amiet, R.K., and Munch, C.L.
United Aircraft Research Laboratories, East Hartford, Conn., J. Aircraft, 12 (1) 34-40 (Jan. 1975), 11 figs, 12 refs

Key Words: aircraft noise, helicopter noise, test models

An experimental investigation was conducted to define the noise characteristics associated with the interaction of a stationary tip vortex and a downstream stationary airfoil. This model test geometry simulated, in its simplest form, the tip vortex-blade interaction occurring on single rotor helicopters during hover. For moderate to high lift test conditions, the vortex-airfoil interaction caused local blade stall with an attendant increase in the blade far-field noise. This interaction may be an important source of helicopter broadband noise during hover. Cross-correlation measurements conducted using surface-mounted and far-field microphones showed that "trailing edge noise" arose from the interaction of stall-generated eddies with the airfoil trailing edge.

76-189

SCATTERING OF OBLIQUELY INCIDENT PLANE ACOUSTIC WAVES BY AN ELASTIC DISK.

Knittel, M.R., Nichols, C.S., and Barach, D.
Naval Undersea Center, San Diego, CA 92132
J. Acoust. Soc. Amer., 58 (5) 983-995 (Nov. 1975) 17 figs, 9 refs

Key Words: wave diffraction, elastic waves

Results are presented for scattering of obliquely incident plane waves from a disk. Total scattering cross sections are calculated as functions of frequency and incident angle. For an incident wave angle of 60°, the calculated displacement distributions on the disk are decomposed into the disk's free vibrational eigenmodes at selected frequencies. The scattering patterns of total pressure and of scattered pressure and contour plots of the nearfield pressure distributions are also presented for certain frequencies.

76-190

ACOUSTIC IMPEDANCE OF AN ANNULAR CAPILLARY

Backus, J.
Univer. of Southern California, Los Angeles, CA 90007, J. Acoust. Soc. Amer., 58 (5) 1078-1081 (Nov. 1975)

Key Words: acoustic impedance, tubes, cylindrical bodies, rods

The behavior of an acoustic impedance formed by a capillary of annular cross section was investigated. A cylindrical rod spaced concentrically within a cylindrical tube was found experimentally to have a relatively constant impedance magnitude up to frequency values for which the length of the capillary was comparable to the wavelength. Computer calculations based on the relevant equations showed that the impedance magnitudes are reasonably constant over a useful frequency range for certain lengths and annulus thicknesses. Graphs of impedance magnitudes and phases are included.

76-191

ENERGY EVALUATION OF WIDE-BAND SOFAR TRANSMISSION

Porter, R.P. and Leslie, H.D.
Woods Hole Oceanographic Institution, Woods Hole, MA 02543, J. Acoust. Soc. Amer., 58 (4) 812-822 (Oct. 1975) 14 refs

Key Words: sound transmission, ray-mode analysis, computer programs

Ray-mode analysis was used to calculate transmitted energy levels for wide-band SOFAR transmission for arbitrary sound-speed

profiles. Computations of wide-band propagation based on ray-mode analysis require an order of magnitude fewer calculations than by harmonic analysis. A computer code has been developed and used to estimate the wide-band loss for profiles from the Mediterranean and the North Atlantic. Comparisons with continuous tone, normal-mode calculations yield agreement to within 3 dB.

76-192

DETERMINATION OF THE WAVE VELOCITY IN AN INHOMOGENEOUS MEDIUM FROM THE REFLECTION COEFFICIENT

Razavy, M.

Theoretical Physics Institute, Dept. of Physics, Univer. of Alberta, Edmonton, Alberta, Canada, J. Acoust. Soc. Amer., 58 (5) 956-963 (Nov. 1975) 7 refs

Key Words: wave propagation, wave diffraction

The velocity of wave propagation in an inhomogeneous medium is obtained from the reflection coefficient (given for all frequencies) for a one-dimensional wave equation. The problem is reduced to determining a "potential" for a Schrodinger-type equation. This "potential," which depends quadratically on the wavenumber, changes the structure of the spectral function and integral equations of the inverse problem. It is first assumed that the wave velocity at plus or minus infinity has the same value. Then this condition on the wave velocity is relaxed, and cases are considered in which the wave velocity at plus infinity is different from the wave velocity at minus infinity with both quantities having well-defined values. For certain types of velocities, the reflection coefficient is obtained analytically and used to find approximate solutions to the inverse problem. An example of a heterogeneous medium which is transparent for all frequencies of the incident wave is presented.

76-193

METHODS FOR DETERMINING CHARACTERISTICS OF ACOUSTIC WAVES IN ROCKET MOTORS

Mathes, H. B. and Price, E. W.

Naval Weapons Center, China Lake, Calif., J. Spacecraft and Rockets, 12 (1) 39-43, (Jan. 1975) 7 figs, 5 refs

Key Words: combustion excitation, rocket engines, test models, computer programs, finite element technique, elastic waves, sloshing

Combustion instability in rocket motors is associated with gas oscillations in one or more standing acoustic wave modes within the motor. The structure of these modes is needed to understand firing data, for proper instrumentation of the motor, and for stability calculations. Two methods for determining standing acoustic solutions to a finite-element program by high-speed digital computer are discussed. Acoustic model results are compared with computer solutions and with motor firing data.

76-194

NATIONAL MEASURE OF AIRCRAFT NOISE IMPACT THROUGH THE YEAR 2000

Final report

Bartel, C., Godby, L., and Sutherland, L.

Wyle Research, El Segundo, Calif., WCR-74-13 EPA/550/9-75-024, 107 pp, (April 1975)

Key Words: aircraft noise, airports, noise reduction

The study was based on the evaluation of operations at Los Angeles International, St. Louis, and Washington Dulles airports. Primary noise reduction alternatives were applied for the years 1987 and 2000. Secondary abatement alternatives were evaluated for 1987 only. The effectiveness of the various alternatives was measured in terms of the total area impacted under the NEF 30 and 40 contours. This area was then increased by a constant factor to obtain an estimate of the impact at the national level. The report also contains an estimate of the total area within the NEF 20, 30, and 40 exclusive of airport property and water.

PB-243 522/0GA

Sponsor: Contract EPA-68-01-2449

76-195

ENVIRONMENTAL NOISE MONITORING AT THREE SITES IN IRVING, TEXAS, NOVEMBER DECEMBER, 1974

Putnicki, G. J. and Watson, H.
Environmental Protection Agency, Dallas, Tex.
Region VI, EPA/906/9-75-001, 67 pp (Feb. 1975)

Key Words: aircraft noise, airports, noise reduction, noise measurement

The results of a study to sample and evaluate noise exposures in a community adjacent to Dallas/Fort Worth Airport where aircraft noise complaints have multiplied are given. The day-night average sound level at three sites was measured for each of seven days using prototype, automatic computerized sound sampling systems and a special interactive tape recording system. Results were spot-checked with conventional equipment.
PB-242 567/6GA

76-196

A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE SCATTERING OF ACOUSTIC WAVES BY RANDOMLY ROUGH SURFACES

Technical report
Welton, P. J.

Texas Univer. at Austin Applied Research Labs, Rept. no. ARL-TR-75-30, 188 pp, (May 1975)

Key Words: underwater sound, acoustic scattering, surface roughness

Scattering of acoustic waves by randomly rough surfaces is studied theoretically and experimentally. A new formulation of scattering integrals, based on the potential method, is developed. Only the expression for the singly scattered fluid is specialized to a form suitable for scattering calculations. The ensemble average of the scattered pressure provides a general result that, except for a proportionality constant, the mean scattered pressure and the probability density function of surface heights are Fourier transform pairs. The mean scattered pressure is calculated for both a Gaussian and a Laplacian probability density function.
AD-A012 866/0GA

76-197

CONSTRUCTION NOISE: SPECIFICATIONS, CONTROL, MEASUREMENT, AND MITIGATION
Final report

Schomer, P. D. and Homans, B.
Army Construction Engineering Research Lab
Champaign, Ill., Rept. no. CERL-TR-E-53, 81 pp (April 1975)

Key Words: construction equipment, noise reduction, noise measurement, specification

This report relates to noise as it affects man, and Army requirements for the prevention of excessive noise. With this background, sample specifications are prepared to control construction-site noise and the means established to monitor compliance. Finally, information is given on state and local noise regulations and on noise-mitigation techniques.
AD-A010 629/4GA

76-198

AIRCRAFT NOISE DEFINITION

Williams, B. G. and Yates, R.
Boeing Commercial Airplane Co, Seattle, Wash.
Rept. no. D6-41302 FAA-EQ-73-7-1, 78 pp, (Dec. 1973)

Key Words: aircraft noise, noise reduction

Acoustic data acquisition, reduction systems and analytical procedures used to derive noise characteristics of a series of Boeing aircraft types are presented. Noise data are presented in EPNdB and db(A) units, from takeoff to low approach thrust and for aircraft altitudes from 200 to 12,000 feet. Areas of possible data deficiency are identified: 90% confidence limits established where possible. A proposed flight test program has the objectives of improving data accuracy and establishing confidence limits over a wide range of operating conditions.
AD-A012 384/4GA
Sponsor: Contract DOT-FA73WA-3254

76-199

**NOZZLE DESIGN FOR THE
ABATEMENT OF BLAST NOZZLE NOISE**
Sneckenberger, J.E. and Pellegrin, J.D.
West Virginia Univer., Morgantown, WV,
ASME paper no. 75-DET-50

Key Words: noise reduction, nozzles

This paper reports on research to reduce the noise level produced by the nozzle of an industrial blast system without appreciably affecting the existing surface improvement performance. Experimental development using noise reduction techniques extrapolated from aerodynamic jet noise abatement methods lead to several improved nozzle designs. The experimental facilities developed for measurement of sound pressure level, abrasive flow rate and abrasive velocity are described.

76-200

**SOUND RADIATION FROM THE IMPACT OF
A SPHERE ON AN ELASTIC PLATE**
Akay, A., Hodgson, T.H., and Bailey, J.R.
N.C. State Univer., Raleigh, NC. ASME
paper no. 75-DET-51

Key Words: sound generation, shock response, spheres, plates

The results of an experimental study made on the sound generated by the impact of small spheres onto steel plates of different thicknesses are reported. Both steel and acrylic spheres, ranging in size from 1.905 to 3.81 cm in diameter, were used in a series of experiments in an anechoic chamber in which the spheres were dropped onto the center of the plate from various heights. Acceleration measurements were also made on the plate to aid in identification of the mechanisms of sound generation. A linear relationship between sound pressure level and plate acceleration level at impact was obtained for each ballplate combination.

76-201

**CORE ENGINE NOISE CONTROL PROGRAM:
VOLUME I. IDENTIFICATION OF
COMPONENT NOISE SOURCES**
Final report
Kazan, S.B., Matta, R.K., Bilwakesh, K.R.,
Harris, V.G., and Latham, D.
General Electric Co., Cincinnati, Ohio Aircraft
Engine Group, FAA-RD-74-125-1, 119 pp,
(Aug. 1974)

Key Words: commercial aircraft, aircraft noise, engine noise, noise generation, noise reduction

The various noise sources constituting the core engine noise for turbofan engines were identified and rank ordered. An investigation was made to ascertain the generating mechanisms, controlling variables, means of identification, and the effect on engine design if reduction were required for each of eight core engine noise sources. The relative significance of the various noise sources was evaluated by predicting the noise contribution of the individual components by the methods derived during the course of the Core Engine Noise Control Program. The predictions were made for each of the three hypothetical cycles for bypass ratios of 4/7, and 14, respectively, which were formulated to encompass a range of commercial aircraft powerplants.
AD-A013 128/4GA

Sponsor: Contract DOT-FA72WA-3023

76-202

**CORE ENGINE NOISE CONTROL PROGRAM,
VOLUME II. IDENTIFICATION OF NOISE
GENERATION AND SUPPRESSION
MECHANISMS**
Final report
Kazin, S.B., Matta, R.K., Bilwakesh, K.R.,
Clapper, W.S., and Emerling, J.J.
General Electric Co., Cincinnati Ohio Aircraft
Engine Group, FAA-RD-74-125-2, 517 pp,
(Aug. 1974)

Key Words: commercial aircraft, engine noise, aircraft noise, noise reduction, noise generation

The mechanisms of noise generation and suppression for the various core engine noise sources in turbofans were defined through a balanced analytical and experimental program. Model, component, and engine tests were used to substantiate the results of the analysis and to determine the basic noise generating parameters. The results were cast in a general form so as to be applicable to a wide variety of cycles, including future technology turbofan engines.

AD-A013 129/2GA

Sponsor: DOT-FA72WA-3023

76-203

CORE ENGINE NOISE CONTROL PROGRAM: VOLUME III PREDICTION METHODS

Final report

Kazin, S.B., Matta, R.K., Bilwakesh, K.R.,
Emmerling, J.J., and Latham, D.

General Electric Co Cincinnati Ohio Aircraft
Engine Group, FAA-RD-74-125-3, 179 pp.
(Aug. 1974)

Key Words: commercial aircraft, aircraft
noise, engine noise, noise reduction, noise
prediction

Prediction methods for core engine noise were reviewed and either updated or new noise evaluation techniques formulated for low velocity coannular jets, combustors ('core' noise). Low pressure turbines, interaction between turbine tones and fan/core jet streams, obstructions in the flow passages and casing radiation. The development was based, to a large extent, on the analytical investigation and the model, component and engine tests evaluated during Phases 2 and 3 of this program. The results were cast in a general form, so as to be applicable to a wide variety of cycles, including present and future turbofan engines. The prediction methods were validated with measured acoustic data wherever possible.

AD-A013 131/8GA

Sponsor: Contract DOT-FA72WA-3023

76-204

AIRCRAFT NOISE GENERATION, EMISSION AND REDUCTION

Lecture series

Advisory Group for Aerospace Research and
Development Paris (France), AGARD-LS-77,
187 pp, (June 1975)

Key Words: aircraft noise, sonic boom, noise
reduction, human response

The physical properties of aircraft noise are summarized. Jet noise and fan-compressor-propeller-rotor noise are emphasized. Topics include acoustic fundamentals, noise source characteristics and interactions, atmospheric propagation, airframe noise, sonic boom, and duct liner and muffler theory. Research and technology activities related to jet engine noise and its control are discussed. The impact of this noise on people, communities, and operational procedures for noise minimization are reviewed.

AD-A012 090/7GA

76-205

ONE-DIMENSIONAL ANALYSES: SHOCK WAVE PROPAGATION FROM UNDERWATER CRATERING DETONATIONS.

Final report

Snell, C.M.

Army Engineer Waterways Experiment Station
Livermore Calif. Explosive Excavation
Research Lab, WES-MP-E-75-1, 566 pp,
(Jan. 1975)

Key Words: shock wave propagation, under-
water explosions, cratering

It has become apparent that the greatest operational advantages regarding the use of large buried chemical explosive charges for engineering excavation may be realized by applying the technique to rock excavation in an underwater environment. Small-scale modeling tests and two new factors that significantly influence underwater cratering processes: (1) early-time dynamic effects caused by the presence of the rock-water interface and water layer; and (2) very late-time water washback and slope failure effects in the crater vicinity. The first of the two effects is studied using hydrodynamic computer calculations.

AD-A012 065/9GA

RANDOM

(Also see Nos. 249, 283, 299, 312)

76-206

TIME-DEPENDENT SPECTRAL CONTENT OF SYSTEM RESPONSE

Corotis, R. B., and Vanmarcke, E. H.
Dept. of Civ. Engrg., Northwestern Univer.,
Evanston, Ill., ASCE J. Engr. Mech. Div.,
101 (EM 5) 623-637 (Oct. 1975) 14 refs, 4 figs

Key Words: random vibration, statistical analysis, probability theory, time-dependent excitation

Random vibration analysis is used to assess the ability of a structure to withstand seismic and wind forces. In the former case the non-stationarity of the mean square value and relative frequency content is important. The concept of a time-dependent frequency domain description of a random process is applied to a linear one degree-of-freedom system suddenly exposed to a zero-mean steady wide-band random excitation. The evolving bandwidth of the oscillator response can be measured by a shape function in terms of the first few spectral moments of the response time-dependent spectral density function. Shape depends on oscillator damping and number of cycles of response. The result can be used to estimate the equivalent viscous damping from recorded structural response to earthquake or wind excitation. Record length and oscillator period and damping affect the reliability of the damping estimate.

SEISMIC

(Also see Nos. 169, 170, 305)

76-207

GENERATION OF SEISMIC FLOOR SPECTRA

Singh, M. P.
Sargent & Lundy, Chicago, Ill., ASCE J. Engr. Mech. Div. 101 (EM 5) 593-607 (Oct. 1975)
8 refs, 6 figs

Key Words: seismic design, spectrum analysis, electric power plants

A simple method for use on digital computers is presented for generating floor response spectra directly from a given ground response spectrum. It is based on transfer characteristics of the structure for random excitation and makes use of the structural frequencies, mode shapes and participation factors, and the prescribed spectrum to generate floor spectra. Use of the proposed method is demonstrated by generating floor spectra of a building.

76-208

GUIDELINES FOR DEVELOPING DESIGN EARTHQUAKE RESPONSE SPECTRA

Final report

Hays, W. W., Algermissen, S. T.,
Espinosa, A. F., Perkins, D. M., and
Rinehart, W. A.

Army Construction Engineering Research Lab
Champaign, Ill., Rept. no. CERL-TR-M-114,
362 pp, (June 1975)

Key Words: seismic design, earthquakes, response spectra

Information for developing design earthquake response spectra provides general guidelines for estimating the ground motion load expected for sites throughout the United States. This document contains a subset of the knowledge available and is considered pertinent to a technical understanding of theoretical and empirical bases currently used to develop design earthquake response spectra for construction, design, and evaluation of facilities. The guidelines relate to the following:

- (1) determination of seismicity parameters,
- (2) estimation of maximum intensity of shaking,
- (3) estimation of seismic attenuation functions,
- (4) estimation of ground motion response spectra, and (5) estimation of local soil amplification effects. Examples are extracted from published reports to demonstrate representative U.S. seismic design problems and use of the guidelines. Extensive references are included.

AD-A012 728/2GA

76-209
**MODAL ANALYSIS METHODS IN SEISMIC
DESIGN FOR BUILDING**

Final report
Stockdale, W.K.
Army Construction Engineering Research Lab,
Champaign, Ill., rept. no. CERL-TR-M-132,
36 pp, (June 1975)

Key Words: modal analysis, seismic design,
earthquake-resistant structures, military
facilities

Changes in tri-services manual TM 5-809-10,
Seismic Design for Buildings are necessary to
provide guidance for the design of critical
military facilities. This report describes
modal analysis methods used in the dynamic
analysis of structures in conjunction with
earthquake response spectra and time history
methods. Elastic and inelastic conditions,
structural damping, and assumptions and
limitations of the methods are described.
Example calculations are included.
AD-A012 732/4GA

76-210
**SAN FERNANDO, CALIFORNIA, EARTHQUAKE
OF FEBRUARY 9, 1971. VOLUME 11. UTILITIES
TRANSPORTATION, AND SOCIOLOGICAL
ASPECTS: ENERGY AND COMMUNICATION
SYSTEMS, WATER AND SEWERAGE SYSTEMS,
TRANSPORTATION SYSTEMS,
SOCIOLOGICAL ASPECTS**

Murphy, L.M.
National Oceanic and Atmospheric
Administration, Boulder, Colo. Environmental
Research Labs, AOAA-75062701, 332 pp
(1973)

Key Words: earthquake damage

This is the second of a three-volume set
documenting engineering and scientific
aspects of the San Fernando earthquake. The
effects of earthquakes are described per-
taining to energy and communication systems,
water and sewage systems, transportation
systems, emergency medical problems,
mass care and assistance, building inspection
and safety, evacuation of unsafe areas, law
enforcement, traffic control, mental health,
aspects, role of the media, changes in
boundary lines and taxable land and improve-

ments, earthquake insurance, voting behavior
and community perspectives on earthquake
problems.

COM-75-10974/4GA

76-211
**COUPLED BENDING AND TORSIONAL
RESPONSE OF NUCLEAR POWER PLANT
STRUCTURES TO EARTHQUAKE LOADING**

Bahar, L.Y. and Ali, S.A.
United Engineers and Constructors, Inc.,
Philadelphia, Pa., ASME paper no. 75-DET-52

Key Words: coupled response, earthquake
damage, nuclear power plants bending,
torsional response, computer programs

A method by which the capability of a general
purpose computer program can be used to
accommodate the coupled bending-torsion
analysis is developed. Its validity is verified
by means of an analytical formulation and
solution of sample problems. Torsional effects
in nuclear power plants arise because of the
nonuniform mass distribution of the equipment
at various floor levels, lack of symmetry of
the building assembly due to auxiliary buildings
or an uneven distribution of the stiffness of
lateral load carrying members.

76-212
**THE SEISMIC ENVIRONMENT FOR NUCLEAR
POWER PLANT COMPONENTS-AN
INTERFACE PROBLEM**

Hadjiam, A.H. Jan H.W. and Linderman, R.B.
Bechtel Power Corp., Norwalk, Calif., ASME
paper no. 75-DET-53

Key Words: nuclear power plants, seismic
design

This paper describes the present state-of-the-
art regarding the interface problems generated
only by the requirements of seismic qualifi-
cation of all safety-related structures, systems,
and components. Recommendations are made
to improve the present situation.

76-213**DETERMINING INTERACTION EFFECTS IN THE SEISMIC ANALYSIS OF COMPONENTS**

Gelman, A. P.

Rockwell International, Canoga Park, Calif., ASME paper no. 75-PVP-50

Key Words: response spectra, seismic response, nuclear power plants

This report presents the development of a simplified method for determining the effect of dynamic coupling (interaction) on components under base excitation (seismic) loading, using the response spectrum technique.

76-214**PIPING SEISMIC RESTRAINT SPACING CRITERIA-AN APPROACH TO THE OPTIMIZATION OF THE USE OF THE SEISMIC RESTRAINTS**

Abdel Sayed, R. K.

Commonwealth Associates, Inc., Jackson, Mich., ASME paper no. 75-PVP-51

Key Words: piping systems, optimization, seismic design

The approach presented in this paper develops and generates piping design spacings to meet the seismic design requirements. Two sets of design spacings, based on stress and frequency criteria, can be developed; one for rigid regions and one for flexible regions. Design guidelines and limitations in the use of rigid and flexible regions are also presented.

76-215**EARTHQUAKE SIMULATOR STUDY OF A REINFORCED CONCRETE FRAME**

Hidalgo, P and Clough, R. W.

Calif. Univer., Berkeley, Earthquake Engineering Research Center, EERC-74-13, 292 pp (Dec. 1974)

Key Words: earthquake resistant structures, concrete construction, testing techniques, dynamic tests, seismic excitation

This study presents the results of the first test of a reinforced concrete structure conducted at the Earthquake Simulator Laboratory located at the University of California, Berkeley. A two story, one bay structure, representing the behavior of a typical small apartment or office building, was designed to meet the ductile concrete requirements of the 1970 Uniform Building Code and the ACI 318-71 Code. Using the 20 ft. square shaking table, it was subjected to a series of simulated earthquake ground motions with intensity large enough to cause significant inelastic deformations and change in the properties of the structure; the maximum base acceleration achieved was 0.46 g. The test structure was then repaired by injecting epoxy into the cracks and tested again with ground motions of equal or larger intensity than before. The structure was finally tested until failure using a static, horizontal load applied at the top floor.

PB-241 944/8GA

SHOCK

(See also Nos. 177, 237, 238, 246, 261, 286, 330)

76-216**NUCLEAR SHOCK WAVE PROPAGATION THROUGH AIR ENTRAINMENT SYSTEMS OF HARDENED FACILITIES**

Final report

Fashbaugh, R. H., Widawsky, A., and Pal, D. Civil Engineering Lab (Navy) Port Hueneme, Calif., CEL-TR-820, 56 pp, (April 1975)

Key Words: shock wave propagation, nuclear explosion damage, underground structures, hardened installations, ducts

The capability of predicting nuclear shock wave propagation through air entrainment systems of hardened facilities was developed. All solutions are one-dimensional. Two finite-difference schemes for approximately the hyperbolic partial differential equations of fluid dynamics are used: the pseudo-viscosity scheme in a variable area Lagrange formulation, and the Lax-Wendroff two-step scheme in an Eulerian formulation. Shock wave attenuation due to viscous losses at ducting walls is included. Comparison of the results of the analysis with shock tube experimental data shows both formulation to be adequate.

AD-A011 812/5GA

76-217

**ONE-DIMENSIONAL BLAST WAVE
PROPAGATION**

Norwegian Defense Construction Service,
Oslo, 29 pp, (Jan. 1974)

Key Words: shock wave propagation, steel,
concrete, rock, underground structures,
ammunition, explosives, storage

A series of experiments is reported on one-dimensional blast wave propagation in steel tubes (with diameters of 5 and 20 cm respectively), in a concrete tube (with a diameter of 80 cm), and in rock (with an effective diameter of 600 cm) with pressures ranging from 0.005 to 70 bar. The blast wave originates from the detonation of TNT charges placed in the center of tubes/tunnel. AD-A010 738/3GA

76-218

**DYNAMIC RESPONSE OF THE M113 GUN
TUBE TO TRAVELLING BALLISTIC
PRESSURE AND DATA SMOOTHING AS
APPLIED TO XM150 ACCELERATION
DATA**

Simkins, T., Pflegl, G., and Scanlon, R.
Watervliet Arsenal, NY, WVT-TR-75015,
112 pp (April 1975)

Key Words: gun barrels, shock excitation,
finite element technique, computer programs

This work is concerned with the response of the M113, 175mm gun tube to the travelling interior ballistic pressure pulse initiated upon firing. The analysis was performed by the NASTRAN finite-element computer code. Initial results indicate that peak dynamic stresses (axisymmetric) can be significantly greater than the static Lamé stresses - normally computed for design purposes. The results compare favorably with strain data accumulated during test firing programs conducted by the Ballistic Research Laboratories at Aberdeen, Maryland. AD-A010 662/5GA

76-219

**A REVIEW OF FACTORS AFFECTING DAMAGE
IN BLASTING**

Ladegaard-Pedersen, A. and Dally, J.W.
Maryland Univer., College Park, Dept. of
Mechanical Engineering, NSF/RA/T-75-016,
179 pp (Jan. 1975)

Key Words: blast effects, shock waves, noise
generation, ground motion

An extensive review is presented of four different characteristics of blasting which can produce damage to inhabitants, vehicles, or structures in urban areas. These include the air blast wave and the noise associated with the venting of the charge, the stress waves propagating into adjacent regions producing ground vibration, and the debris ejected from the crater or bench face when the bore hole is over-charged. Each of these four different features which can produce damage is treated separately. Fundamental relations are reviewed in each case and, where possible, explicit theoretical results are presented in terms of scaled parameters which permit predictions to be made for any size charge. PB-242 224/4GA

PHENOMENOLOGY

DAMPING

(Also see Nos. 161, 230 253, 285)

76-220

**CONTROL OF GAS TURBINE STATOR
BLADE VIBRATIONS BY MEANS OF
ENAMEL COATINGS**

Jones, D.I.G. and Cannon, C.M.
Air Force Materials Laboratory, Wright
Patterson Air Force Base, Ohio, J.
Aircraft 12 (4) 226-330 (April 1975) 13 figs,
4 refs

Key Words: turbine components, gas turbine
blades, blades, vibration control, vibration
damping, coatings

A high-temperature enamel coating was applied to part of the surface area of the stator vanes of a jet engine to increase damping. Such a coating could increase the damping level. Many of the engineering problems that must be overcome before all high-temperature vibrations will be controllable are described.

76-221

ENCAPSULATED TUNED DAMPERS FOR JET ENGINE COMPONENT VIBRATION CONTROL

Parin, M. L. and Jones, D. I. G.
Univer. of Dayton, Dayton, Ohio, J.
Aircraft 12 (4) 293-295 (April 1975) 3 figs,
5 refs

Key Words: tuned dampers, jet engines,
vibration control

Serious vibrational resonance problems are induced in jet engine components by the engine environment. A preliminary investigation of a tuned damper, utilizing damping material encapsulation to withstand centrifugal load and erosion problems was made. Tests were conducted at room temperature for convenience; results are applicable to higher temperatures if the appropriate damper materials are used. Tests on one mode of a turbine blade showed a great reduction in vibration levels resulting from proper tuning of the damper.

76-222

DYNAMIC DAMPING OF WIND-INDUCED STOCHASTIC VIBRATIONS

Akesson, B. A.
Chalmers Univer. of Technology, Gothenburg,
Sweden, ASME paper no. 75-DET-10

Key Words: vibration absorption (equipment)
wind-induced excitation

The internal stress resultant in a lightly damped linear structure exposed to gusty natural wind is calculated. Methods to reduce this stress resultant are studied. The inefficiency of raising the natural frequency of the structure by increasing its stiffness is shown. The efficiency and the stability of a suitably tuned and damped small dynamic vibration absorber as installed in the wind-loaded structure are investigated.

76-223

TUNABLE, NON-LINEAR VIBRATION ABSORBER

Miller, H. M. and Gartner, J. R.
The Univer. of Connecticut, Storrs, Conn.,
ASME paper no. 75-DET-9

Key Words: vibration absorption (equipment),
pneumatic springs, periodic excitation

A tunable, nonlinear, "hard" spring vibration absorber acting in conjunction with a lightly damped linear system experiencing forced sinusoidal excitation, subjected to both theoretical analysis and experimental verification, is discussed. The specific "hard" spring used is a pneumatic spring of constant cross-sectional area, governed by the polytropic gas laws. Design details are given for a prototype device that successfully realized the governing principles.

FATIGUE

(Also see No. 176)

76-224

DISTORTION-INDUCED VIBRATION IN FAN AND COMPRESSOR BLADING

Danforth, C. E.
General Electric Co., Cincinnati, Ohio, J.
Aircraft, 12 (4) 216-225 (April 1975) 14 figs,
22 refs

Key Words: fans, blades, compressor blades,
fatigue

Eight mechanisms of distortion-induced vibration are described, only one of which has previously been discussed in the literature. Distortion characteristics are relevant for assessing blade resonance, random vibration in separated flow, and flutter. A distortion index for blade vibration is defined.

FLUIDS

(Also see No. 205)

76-225

SURFACE WAVE MODES ON ELASTIC CYLINDERS

Frisk, G.V., Dickey, J.W., and Uberall, H. Naval Research Laboratory., Wash. DC 20375, J. Acoust. Soc. Amer., 58 (5) 996-1008, (November 1975) 10 figs, 20 refs

Key Words: surface vibration, mode shapes, submerged structures

The surface wave modes that may exist on solid elastic cylinders embedded in a fluid are close to either the wave speed in the fluid (Stoneley-and Franz-type modes) or to the bulk wave speeds in the solid (Rayleigh- and Whispering Gallery-type modes). Analytic and numerical methods were used in a discussion of the modes. In the limit of infinite cylinder radius, the wavenumbers of the Rayleigh and Stoneley modes tend toward those of the Rayleigh and Stoneley waves on a flat elastic half-space; the Franz mode wavenumbers tend toward the wavenumber of sound in the ambient fluid. The sheet structure of the elastic surface waves is discussed in an appendix.

76-226

FLUID FORCES INDUCED BY VORTEX SHEDDING

Blevins, R. D. and Burton, T. E. General Atomic Co., San Diego, Calif., ASME paper no. 75-FE-10

Key Words: mathematical models, vortex shedding, fluid-induced excitation, cylinders

A semi-empirical, dynamic model for investigating the fluid forces induced on a bluff cylinder by vortex shedding is described using random vibration theory. The model includes both spanwise correlation effects and the amplitude dependence of the correlated vortex forces. Model parameters are determined by experimental data. The results are then applied to determine the forces exerted on elastic cylinders at resonance with vortex shedding. The predictions are in good agreement with experimental data.

76-227

EXPERIMENTAL AND ANALYTICAL STUDY OF LIQUID AND TWO-PHASE FLOW INDUCED VIBRATION IN REACTOR FUEL BUNDLES

Gorman, D.J. Univer. of Ottawa, Ottawa, Canada, ASME paper no. 75-PVP-52

Key Words: experimental data, testing techniques, fluid-induced excitation, cylindrical shells, nuclear fuel elements

Vibration tests on an instrumented cylindrical tube simulating a reactor fuel pin were conducted. Tests were conducted with two-phase air-water mixtures and liquid flow. The objective was to establish the effects of mass flow rates and void fractions on the amplitude of vibration.

76-228

SAMPLING STATISTICS FOR CYLINDRICAL MODES OF HIGHER ORDER

Jones, K. E. and Waterhouse, R. Naval Ship Research and Development Center, Bethesda, MD 20084, J. Acoust. Soc. Amer., 58 (4) 846-852 (October 1975) 3 refs

Key Words: statistical analysis, axisymmetric vibration, fluids

Distribution functions for random sampling in space of a three-dimensional acoustic mode were considered. Expressions were obtained for the probability density and cumulative functions for a single mode excited in an ideal elastic fluid contained in a right-circular cylindrical enclosure. The work was extended to axisymmetric modes of higher order; rms values and mean-square values of the modal function were sampled. The rms modal function contained cusps on the abscissa that caused the distribution functions to differ from those pertaining to the square of the modal function. In both cases, however, the density functions had a number of poles that increased with the order of the mode. Computed values of the functions and values of the variances of the distributions are presented.

76-229
HARMONIC MOTION OF CYLINDERS IN
UNIFORM FLOW

Fry, J. T.
Naval Postgraduate School, Monterey, Calif.
53 pp. (June 1975)

Key Words: fluid induced excitation, cables,
cylinders

The time-dependent force acting on a cylinder undergoing harmonic in-line oscillations in an otherwise steady flow was measured for various amplitudes and frequencies of oscillation and mean flow velocity. The experiments were carried out in a recirculating water tunnel operating as an open channel with a free surface at the test section. The time-dependent force has been expressed in terms of a mean drag coefficient and the Fourier-averaged drag and inertia coefficients and as functions of the relative amplitude and a frequency parameter.
AD-A012 412/3GA

VISCOELASTIC

76-230
A FINITE ELEMENT COMPUTER PROGRAM
FOR PREDICTING THE NONLINEAR
STATIC AND DYNAMIC BEHAVIOR OF
VISCOELASTIC COMPONENTS

Dieterich, D. A.
Structural Dynamics Research Corp.,
Cincinnati, Ohio. ASME paper no 75-DET-7

Key Words: finite element technique,
computer programs, shock isolation,
vibration isolation, viscoelastic damping

The Viscosuperb computer program described in this paper used the popular parabolic isoparameter solid finite element to represent the viscoelastic component (with frequency and temperature dependent material properties) in both large deflection static analysis for preloaded position and linearized dynamic analysis for stiffness and damping. The finite element approach has recently been extended to handle the large-deflection nonlinear behavior of viscoelastic/structure assemblies.

76-231
WAVE PROPAGATION IN A VISCOELASTIC
ROD WITH TEMPERATURE-DEPENDENT
PROPERTIES

Ting, E. C.
School of Civil Engineering, Purdue Univer.,
West Lafayette, IN 47906, J. Acoust. Soc.
Amer., 58 (5) 1018-1022 (November 1975)
2 figs, 20 refs

Key Words: wave propagation, rods,
viscoelastic media, temperature effects

The unidirectional wave propagation in a rod made of viscoelastic material of the Maxwell type is considered. The rod is subjected to nonuniform temperature gradients and its mechanical properties are assumed to be temperature dependent. A perturbation technique incorporated with the usual Laplace transform method is used to obtain the analytical solution in series form.

EXPERIMENTATION

DATA REDUCTION
(Also see No. 318)

DIAGNOSTICS

76-232
VIBROACOUSTIC DIAGNOSTICS AND THE
REDUCTION OF THE VIBRATION OF
SHIPOBOARD MACHINERY

Popkov, V. I.
Joint Publications Research Service,
Arlington, VA., 238 pp. (June 6, 1975)

Key Words: shipboard equipment response,
machinery noise, machinery vibration,
vibration isolation

The contents of this report are: Vibration levels and the dynamic interaction of shipboard machinery with supporting and nonsupporting links; vibrational energy flux to the supporting and nonsupporting links during operation of shipboard machinery; methods

and means of investigating the vibrations of shipboard machines; vibrational diagnostics of machinery by the data on mechanical resistances, dynamic forces and emitted vibrational energy; decreasing the vibrations in the structural elements of shipboard machinery. JPRS - 64931

EQUIPMENT

(See also Nos. 261, 288, 299, 319)

EXPERIMENT DESIGN

(Also see No. 278)

FACILITIES

(Also see Nos. 215, 297, 313)

76-233

IMPROVEMENT OF SHOCK SPEED IN AN ELECTROTHERMAL DIAPHRAGM SHOCK TUBE

Phillips, M.G.R. and Pugatschew, A.A. School of Physical Sciences, The Flinders Univer. of South Australia, Bedford Park, South Australia 5042, J. Phys. E(Sci. Instr.) 8 (11) 913-914 (Nov. 1975) 3 figs, 7 refs

Key Words: shock tubes, shock wave propagation

The speed of shock waves produced in an electrothermal diaphragm shock tube can be greatly increased without additional energy expenditure by dividing the capacitor bank into two sections each of which is separately discharged. There is an optimum time delay between firing the two capacitor banks. This technique, applied to a small shock tube, raised the shock speed in argon at 2 Torr from Mach 16 to Mach 23.

INSTRUMENTATION

76-234

A VERSATILE SYSTEM FOR THE MEASUREMENT OF INTERNAL FRICTION IN A TORSION PENDULUM

Bleasdale, P.A. and Bacon, D.J. Dept. of Metallurgy and Materials Science, Univer. of Liverpool, PO Box 147, Liverpool L69 3BX. J. Phys. (Sci. Instr.) 8 (6) 467-468 (June 1975) 3 figs, 7 refs

Key Words: measuring instruments, internal friction, natural frequency, pendulum

A system based on automatic measurement of the number of oscillations of a reflected light beam between two predetermined but variable amplitudes is described for measuring the internal friction of a wire specimen in free decay in an inverted torsion pendulum. The system is comparatively inexpensive to produce, is semiautomatic, enables accurate and rapid determination of internal friction and resonance frequency, and has a very wide frequency range.

76-235

STUDY OF THE MEASUREMENT OF TRANSDUCER DIRECTIVITY WITH A PLANAR MEASUREMENT ARRAY

Baker, D.D. and Smith D.A. Applied Research Lab., The Univer. of Texas at Austin, Austin, TX 78712, J. Acoust. Soc. Amer., 58 (4) 807-811 (Oct. 1975) 3 refs

Key Words: sonar transducers, measuring instruments, measurement techniques

An experimental study was conducted to investigate directivity measurements made in the farfield of a conventional sonar transducer with a planar array of probe transducers. The principal questions investigated were (1) spacing of measurement probes so that directivity measurements are of useful accuracy, and (2) useful processing and display of data. Full-scale experiments were conducted with a probe transducer positionable over a grid of measurement points to simulate a multiprobe array. Measurements were made at probe spacings varying from 0.1 to 0.4 times the half-power beamwidth of the sonar transducer. Data were processed by computer and displayed with contour plots. The data obtained from the experimental measurements were compared with conventional beam patterns of the transducer. Discrete data taken at 0.1 and 0.2

times half-power beam width defined directivity function to within the measurement accuracy of conventional directivity measurements. Data obtained at the spacing equal to 0.4 times the half-power beam width, resolved major characteristics of the pattern to within +2 dB and +2. Use of more sophisticated interpolation techniques in processing data prior to contour plotting would probably improve the accuracy.

76-236
APPARATUS FOR MEASURING THE
TORSIONAL MODULUS AND DAMPING OF
SINGLE CARBON FIBRES

Adams, R. D. and Lloyd, D. H.
Dept. of Mech. Engineering, Univer. of
Bristol BS8 1TR, J. Phys. E(Sci. Instr.)
8 (6) 475-480 (June 1975) 6 figs, 9 refs

Key Words: measuring instruments, shear modulus, vibration damping, fibres, pendulum

An apparatus suitable for measuring the dynamic torsional modulus and damping of single filaments at surface strains up to 10^{-3} is described. The major contribution to extraneous losses is aerodynamic damping even at pressures as low as 10^{-6} Torr. For free molecule flow, aerodynamic damping is directly proportional to pressure. The constant of proportionality predicted, however, is at least 28% less than that obtained experimentally; the discrepancy has not yet been explained. The experimental value can be used to allow for aerodynamic losses. It is not necessary to use very low pressures if only the fiber modulus is required.

SCALING AND MODELING

(Also see Nos. 217, 285)

76-237
INVESTIGATION OF UNDERGROUND
EXPLOSIONS WITH MODEL TESTS.
PRELIMINARY REPORT 1.

Schmidt, K. G.
Norwegian Defence Research Establishment
Kjeller, Rept. no. NDRE-VM-182, 38 pp
(March 1975)

Key Words: underground explosions, test models

A preliminary report describing main results at the present stage of an investigation of underground explosions with model tests in the scale 1/100 is presented. Pressure data from measurements using the models are given.
AD-A010 742/5GA

76-238
MODEL TESTS OF ACCIDENTAL
EXPLOSIONS IN UNDERGROUND AMMUNITION
STORAGE: I CHAMBER PRESSURE

Norwegian Defence Construction Service Oslo
28 pp (January 1974)

Key Words: underground explosions, test models, ammunition, storage, explosives, underground structures

The report is a short version of a second report in a series of five describing the results from an extensive series of model tests on underground ammunition storage. The main objectives of the experiments discussed in this paper were to determine the pressure-time history from an explosive charge detonated in chambers varying: loading density, venting, type of explosive (TNT, PETN, AN/FO, RDX, ALUMIT, DYNAMITE and COMP. B) and to compare results with theoretical work and other experiments.

AD-A010 737/5GA

TECHNIQUES

(Also see Nos. 215, 244 317, 324)

76-239
RECORDING TECHNIQUES FOR HIGH SPEED
SHOCK WAVE PASSAGE MONITORED BY A
MULTISENSOR ARRAY

Saint-Hilaire, G., Guay, J. M., and Dimoff, K.
INRS-Energie, Universite du Quebec, D. P.
1020 Varennes Quebec, JOL 2PO Canada
J. Phys. E(Sci. Instr) 8 (4) 277-280 (April
1975) 5 figs, 6 refs

Key Words: measurement techniques, shock waves

Undistorted waveforms from sequentially activated sensing devices were registered on a single wavetrain by delayed blocking of each output signal. A scaling method and fast sweep technique allowed the results of a large number of measuring stations to be recorded on

the screen of one dual beam oscilloscope with no loss of information. The total display time could be up to ten times the duration of one cross screen sweep, depending on the amplitude of the scaled signal.

76-240

**SOME TUNNEL-WALL EFFECTS ON
TRANSONIC FLUTTER**

Ruhlin, C. L., Destuynder, R. M. and Gregory, R. A.
NASA Langley Research Center, Hampton, Va.
J. Aircraft 12 (3) 162-167 (March 1975) 8 figs.
10 refs

Key Words: wind tunnel tests, flutter, testing techniques

Significant effects of wind-tunnel walls were observed on transonic flutter boundaries of wall-mounted models during two flutter model research studies. Flutter experiments with cantilevered SST-type wing models were conducted in the ONERA S2 tunnel at Modane, France; the NASA Langley transonic dynamics tunnel; and the NASA Ames 6-ft. by 6-ft. supersonic tunnel. Experimental results indicated that transonic flutter boundaries can be affected by tunnel-wall interference, tunnel resonances, and shock-wave reflections, and that flutter model data accuracy is a function of model/tunnel size and tunnel wall porosity. A flutter trend analysis for a two-dimensional wing demonstrating tunnel wall and resonance effects on flutter is also presented.

76-241

**FORCED-OSCILLATION TEST MECHANISM
FOR MEASURING DYNAMIC-STABILITY
DERIVATIVES IN ROLL**

Burt, G. E.
ARO, Inc., Arnold Air Force Station, Tenn.
J. Aircraft 12 (1) 11-17 (January 1975)
17 figs, 6 refs

Key Words: testing techniques, aerodynamic stability, aircraft

A test mechanism and associated instrumentation for measuring rolling moment, yawing moment, and side force due to roll velocity on lifting configurations are described. Models with a cavity diameter of two in. can be

installed on the water-cooled test mechanism which can support 1,200 lb normal force and 300 lb axial force. The system can accurately measure dynamic moments of 0.05 in.-lb. and simultaneously provides five-component static force and moment data.

76-242

**THE MEASUREMENT OF FREQUENCY AND
FREQUENCY STABILITY OF PRECISION
OSCILLATORS**

Technical note
Allan, D. W.
National Bureau of Standards, Boulder, Colo.
Time and Frequency Div., NBS-TN-669, 34 pp
(May 1975)

Key Words: measurement techniques, frequency meters, oscillators

This paper presents at the tutorial level some convenient methods for measuring frequencies and/or frequency stabilities of precision oscillators. Advantages and disadvantages of the methods are included.

COM-75-10918/1GA

76-243

**USE OF IMPEDANCE DATA FOR AVOIDING
PROPELLER FAN RESONANCE**

Baade, P. K. and Morris, R. D.
Carrier Corp. Syracuse, N. Y. ASME paper
no. 75-DET-54

Key Words: testing techniques, standards and codes, fans

The results of a project sponsored by the American Society of Heating, Refrigerating, and Air Conditioning to develop a standard method of testing propeller fans used in heating, refrigerating, and air conditioning equipment to measure those dynamic characteristics essential to the proper selection and application of such fans from a standpoint of reliability are described. The purpose of this paper is to discuss what tests should be covered by such a standard and what problems exist in such measurements.

HOLOGRAPHY

76-244

**AERONAUTICAL ANALYTICAL REWORK
PROGRAM: ACOUSTICAL HOLOGRAPHY
SYSTEM DEMONSTRATION ON A-6 WING
SKIN STIFFENER ACOUSTIC IMAGE
INSPECTION**

Interim report

Holosonics Inc., Richland Wash., 27 pp
(May 28, 1975)

Key Words: acoustic holography, aircraft,
nondestructive testing

The objective was to demonstrate the applicability of acoustical imaging techniques using the System 200 Acoustical Holography Inspection System for naval aircraft, in particular the A-6 wing skin stiffener. The successful results obtained reflect the capability of the System 200 Acoustical Holography Inspection System to provide repeatable evidence of the integrity of structures such as the A-6 wing skin stiffener.
AD-A012 584/9GA

76-245

ACOUSTICAL HOLOGRAPHY

Brenden, B.B.

Holosonics Inc., Richland, Wash., J. Phys.
E(Sci. Instr.) 8 (11) 885-894 (November 1975)
17 figs, 53 refs

Key Words: acoustic holography

The various holograph detection schemes are benefiting medical diagnosis, industrial testing, undersea search, and geological exploration. The two most successful detectors have been the liquid surface, which provides instantaneous real time imaging at framing rates in excess of 100 frames/s, and scanned piezoelectric receivers, which have been used extensively for pressure vessel inspection.

76-246

**APPLICATION OF HOLOGRAPHY TO A
STUDY OF WAVE PROPAGATION IN ROCK**
Holloway, D.C. and Patacca, A.M.
NSF/RA/T-75-017, 40 pp (January 1975)

Key Words: explosion effects, rock,
holographic techniques

Double exposure holography using a pulsed ruby laser was employed to record the surface displacements in westerly granite rock models of a half space. The explosive excitation was produced by 100 to 200 mg of lead azide on the free surface of the specimens. Mirrors were used so that three widely separated views of the surface were obtained thus allowing for the solution of the displacements. A radial line from the explosive center was analyzed in detail and the solution was correlated with theory.

PB-242 366/3GA

76-247

**HISTORY AND PRESENT STATUS OF LIQUID
SURFACE ACOUSTICAL HOLOGRAPHY**

Brenden, B.B.

Holosonics, Inc. Richland, WA 99352, J.
Acoust. Soc. Amer., 58 (5) 951-955 (November
1975)

Key Words: acoustic holography

Use of the liquid surface as a detector in acoustical holography began early in 1964. The dynamic response of the liquid surface permits real-time viewing of the image at frame rates of more than 100 per second. Color translation has been demonstrated but is not currently being used in practical systems. Detailed images of soft tissue structures in the hand, arm, and excised organs show tendons, muscles, tendon attachments to the skeletal structure, veins, and arteries.

COMPONENTS

SHAFTS

(Also see No. 163)

vibrations, rotors, gears

A high-speed, gear-coupled, branched rotor system that sustained large lateral vibrations of the high-speed branch at a subharmonic frequency equivalent to the running speed of the low-speed branch of the system is described. Individual torsional and lateral critical-speed calculations for the system were used to infer the torsional-lateral coupled cause of the difficulty.

BEAMS, STRINGS, RODS

(Also see Nos. 164, 229)

76-249

NONLINEAR STRESSES AND DEFLECTIONS OF BEAMS SUBJECTED TO RANDOM TIME DEPENDENT UNIFORM PRESSURE

Seide, P.

Dept. of Civil Engineering, Univer. of Southern Calif., Los Angeles, CA 90007, Israel J. Tech., 13 (1-2) 143-151 (1975)

4 figs, 15 refs

Key Words: beams, random excitation, time-dependent excitation

The nonlinear mean-square multi-mode response of beams subjected to uniform pressure uncorrelated in time was investigated. The method of equivalent linearization was used to obtain mean-square stresses and displacements in beams with arbitrary end conditions. Calculations were carried out for beams with both ends either simply supported or clamped, for the case of white noise excitation. Although the maximum displacement can be obtained with the use of only a single degree of freedom model, it is necessary to consider as many as 100 modal functions for accurate determination of the stresses. The maximum mean-square deflection of the clamped beam was found to be somewhat less than the simply supported

LOADS

Kounadis, A.N.

Ing.-Arch. 44 (1) 43-51 (March 1975) 5 figs, 3 refs

Key Words: beams (supports), columns (supports), variable cross section, harmonic excitation, flexural vibrations, rotatory inertia effects, transverse shear deformation effects

An exact closed-form solution of the governing equations for transverse vibrations of a beam-column of stepwise varying cross section, subjected to lateral harmonic loads is presented. Included are the effects of rotatory inertia and shear-deformation. Simple formulas are given for calculating the dynamic flexibility matrix of the beam-column, its deflection, moments, and shearing forces at the ends of each of its segments without having to determine the natural frequencies of the beam. However, the latter can be readily determined by numerical solution of the frequency equation.

76-251

NATURAL VIBRATIONS OF SUSPENSION CABLES

West, H. H. Geschwindner, L. F., and Suhoski, J. E.

Assoc. Prof. of Civ. Engrg., Pennsylvania State Univer., University Park, Pa., ASCE J. Struc. Div., 101 (ST 11), 2277-2291 (November 1975)

Key Words: natural frequencies, mode shapes, cables (ropes)

Natural frequencies and modes of vibration for suspension cables were determined using a discretized system consisting of a linkage of straight bars connected by frictionless pins having concentrated masses at the connection points. Governing equations were linearized, limiting the applicability to small oscillations. The frequencies were

determined by coupling a generalized Holzer method with the solution of the associated boundary value problem as a set of initial-value problems. The method was applied to sample numerical problems and the results compared with those obtained by alternate approaches. The sensitivity of the results to the nature of the discretization were studied. Several parameter studies were used to determine how the natural vibrations were altered in response to dimensional variations in the suspension cable. The results of the parameter studies are summarized in non-dimensional form.

76-252

DYNAMIC RESPONSE OF CANTILEVERS WITH ATTACHED MASSES

Kounadis, A. N.

National Technical Univer. of Athens, Athens, Greece, ASCE J. Engr. Mech. Div. 101 (EM 5) 695-706 (October 1975) 1 figs, 6 refs

Key Words: cantilever beams, forced vibration

Generalized functions are used to establish in matrix form the frequency equation for free vibrations of a cantilever beam-column having rotational and translational springs at its support, and carrying concentrated masses; the frequency equation is evaluated numerically. The effect of rotatory inertia of the concentrated masses is also included. A procedure for establishing the mode shapes is presented. The differential equation for the modal amplitudes of forced motion is established.

76-253

COUPLED VIBRATION OF ELASTIC CIRCULAR BARS IN VISCOUS FLUID

Shimogo, T., Niino, T., and Setogawa, S. Keio University, Yokohama, Japan, ASME Paper No. 75-DET-76

Key Words: coupled response, bars, viscous damping

The dynamic behavior of two elastic circular bars vibrating in a viscous fluid is investigated with emphasis on interaction between bars. The effects of fluid viscosity on the virtual masses and viscous forces of two bars

are evaluated, and the coupled natural frequencies of the bars and the peak value of frequency responses are determined by linearizing the Navier-Stokes equations under the assumption of small amplitude of response.

BEARINGS

76-254

APPLICATIONS FOR EARLY DETECTION OF ROLLING-ELEMENT BEARING FAILURES USING THE HIGH-FREQUENCY RESONANCE TECHNIQUE

Darlow, M. S., and Badgley, R. H.

Mech. Tech. Inc., Latham, N. Y., ASME Paper No. 75-DET-46

Key Words: anti-friction bearings, high frequency resonance technique, damage prediction

The high-frequency resonance technique (HFRT) has been demonstrated to be a highly sensitive, accurate procedure for the early identification of impending rolling-element bearing failures. The theory behind the HFRT, and its application to rolling-element bearing defect analysis are discussed.

76-255

DYNAMIC BEHAVIOR OF HYDROSTATIC THRUST BEARING

Aoyama, T., Inasaki, I., and Yonetsu, S., Keio University, Yokohama, Japan, ASME Paper No. 75-DET-2

Key Words: hydrostatic bearings, thrust bearings, dynamic response

The purpose of this paper is to investigate the behavior of a hydrostatic thrust bearing, which has four sector-shaped recesses, when a dynamic force and moment are applied on it. By applying the finite element method, dynamic characteristics of the hydrostatic thrust bearing are obtained theoretically.

BLADES

(Also see No. 224)

76-256
ADVANCES IN FAN AND COMPRESSOR BLADE FLUTTER ANALYSIS AND PREDICTIONS
Mikolajczak, A. A., Arnoldi, R. A., Snyder, L. E., and Stargardter, H.
Pratt & Whitney Div., United Aircraft Corp., East Hartford, Conn., J. Aircraft, 12 (4), pp 325-332. (April 1975), 15 figs, 14 refs

Key Words: fans, blades, compressor blades, flutter

A unified approach to flutter prediction has been developed at Pratt & Whitney Aircraft. The aeromechanical stability of the blade-disk system is expressed in terms of a stability parameter that measures the amount of unsteady work done by air on the system vibrating in a natural mode. An accurate prediction of vibrational deflections and of unsteady aerodynamic forces is required at every spanwise location on each blade in order to calculate the work done by the unsteady aerodynamic forces. Recent progress is described in the prediction of unsteady aerodynamic forces and the determination of mode shapes. The stability model is applied to the prediction of supersonic flutter, chordwise bending flutter, and stall flutter. Recommendations are made for improving the prediction model.

76-257
BLADE VIBRATION: SOME KEY ELEMENTS IN DESIGN VERIFICATION
Danforth, C. E.
General Electric Co., Cincinnati, Ohio
J. Aircraft, 12 (4), pp 333-342. April 1975, 14 figs, 5 refs

Key Words: blades, vibration response, design techniques

Two aspects in design verification of blade vibration were studied: assessment at a given operating point of compressor and fan blade vibration in relation to high-cycle fatigue;

and the identification of the worst engine operating points for blade vibration. Design and design verification significance of blade dynamics and three-dimensional stress distributions and levels derived from precision experiments and numerical analysis investigations were illustrated. Illustrative examples are also given for blade vibration, both stable and self-excited.

CYLINDERS

(Also see No. 226)

FRAMES

76-258
OPTIMAL DESIGN OF ELASTIC STRUCTURES UNDER DYNAMIC LOADS
Technical report
Feng, T.T., Arora, J.S., Haug E.J.
Iowa Univer. Iowa City Div of Materials Engineering
AC-CR-75-006 140 pp (May 1975)

Key Words: dynamic response, structure members, trusses, frames, optimum design, steepest descent method

Optimal design of elastic structures under dynamic loads is considered. A general objective functional for the problem is defined, and a functional treatment of the transient dynamic response constraints (stress and displacement constraints) is presented. Constraints on the natural frequency of the structure and the design variables are included. The general method is applied to truss-frame structures.
AD-A012 212/7GA

76-259

DYNAMIC SNAP-THROUGH BUCKLING OF SHALLOW ARCHES WITH NONUNIFORM STIFFNESS

Rapp, I. H., Smith, C. V., and Simitzes, G. J.
Owens/Corning Houston Tex., ASME paper
no. 75-DET-41

Key Words: arches, snap through problems,
dynamic buckling, variable material properties

This paper presents the geometrically non-linear response with emphasis on snap-through buckling of shallow arches with nonuniform stiffness under dynamic loading. The arch is a half-sine pinned arch with symmetric stiffness and mass distribution.

GEARS

(Also see No. 248)

76-260

AN EXPERIMENTAL STUDY OF SOUND-DAMPING RINGS FOR GEARS-DYNAMICAL BEHAVIOR AND OPTIMUM DESIGN PARAMETERS FOR SOUND-DAMPING RINGS

Okamura, H., and Suzuki Y.
Sophia Univer., Tokyo, Japan, ASME paper
no. 75-DET-4

Key Words: noise reduction, gears rings

The dynamic behavior and the damping mechanism of the sound damping rings was investigated by inserting snap rings into the rims of a pair of spur gears. By the insertion of appropriate rings, the gear damping was increased by a factor of about ten. The gear noise was reduced about 5 db in the A-weighted sound level, and the sound and vibration spectral lines corresponding to the gear natural frequencies were reduced. The relative motion between the gear and rings was measured. A simple vibration model for the gear and rings was also developed.

ISOLATORS

(Also see No. 232)

76-261

UNIVERSAL SHOCK AND VIBRATION MOUNTS

Nycum, J. M.
Dept. of the Navy Washington, DC
11 pp (January 1974)

Key Words: shock absorbers, vibration isolation, equipment response, equipment mounts

The patent application relates to a universal mounting base for protecting fragile equipment against shock and vibration. A resilient plastic foam pad of rectangular outline is divided into quadrature air cavities and sandwiched between bottom and top plate. The top plate includes four matrices of holes, each matrix registering with a respective cavity, and being selectively blocked by cover pads interfaced between the top plate and the bottom of the equipment to be supported. Holes selected for blocking are empirically determined for a given weight and center of gravity of the equipment to achieve maximum dynamic stability and protection.
PAT-APPL-435 786/GA

76-262

COMPOUND MOUNTING SYSTEMS THAT INCORPORATE DYNAMIC VIBRATION ABSORBERS

Snowdon, J. C.
Penn. State Univer., Univer. Park Applied
Research Lab., 39 pp (May 1975)

Key Words: mounts, shock absorbers,
dynamic vibration absorption (equipment)

The transmissibility across novel compound or two-stage mounting systems is discussed, and representative calculations of transmissibility are compared for systems with intermediate stages that comprise either (1) a rigid mass plus a dynamic vibration absorber, (2) dual beams loaded symmetrically by equal lumped masses, or (3) dual beams to which multiple dynamic vibration absorbers are attached. In the first case, the undesirable secondary resonance of the compound system can be suppressed effectively by the dynamic vibration absorber, for which design data are given.

In the latter cases, lumped masses or dynamic vibration absorbers are applied to each intermediate beam of the compound system at the locations of the four upper mounts that support the vibrating item from below. In turn, the beams are supported at each end by single antivibration mounts on a rigid foundation.

AD-A011 422/3GA

MECHANICAL

(Also see Nos. 168, 329)

76-263

MOTION RESTRAINING DEVICE

Ford, A.G.

National Aeronautics and Space Administration
Pasadena Office, Calif.. NASA-CASE-NOP-13619-1, 18 pp (April 1975)

Key Words: brakes (motion overstress), energy dissipation, shafts (machine elements)

A motion-restraining device for dissipating at a controlled rate, the force of a moving body was developed. The device is characterized by a drive shaft adapted to be driven in rotation by a moving body and employs a three-stage motion-multiplying gear train. The force of a moving body may be without resorting to the use of conventional motion-control devices such as escape wheels, two arm pallets, and comparable components.
PAT-APPL - 572 990/GA

76-264

HYDRAULIC NOISE STUDY

Oklahoma State Univer.. Stillwater Fluid Power Research Center, OSU-FPRC-4M2 425 pp. (December 1974)

Key Words: hydraulic presses, hydraulic equipment, hydraulic valves, noise reduction

This report presents the results of a hydraulic noise study which experimentally examined the sound power of selected military components and investigated the effectiveness of noise control techniques for fluid power systems.

AD-AO11 170/8GA

PANELS

76-265

VIBRATION AND BUCKLING OF CROSS-PLY LAMINATED CIRCULAR CYLINDRICAL PANELS

Sinha, P.K. and Rath, A.K.

Vikram Sarabhai Space Centre Trivandrum, India, Aeronaut. Quart. 26 (3) 311-218 (Aug. 1975) 5 figs 16 refs

Key Words: free vibration, stability, cylindrical shells, panels

The free transverse vibration and stability analysis of circular cylindrical composite panels was studied. The panels were assumed to be composed of an arbitrary sequence of 0 and 90 layers. Non-dimensional vibration and buckling parameters were computed for simply-supported panels consisting of antisymmetric cross-ply graphite-epoxy laminates. Discussions are included on the influence of the coupling between bending and extension, shear deformation, the panel curvature, and the aspect ratio.

PIPES AND TUBES

(Also see No. 227)

76-266

EFFECTS OF THE STEAM CHEST ON STEAMHAMMER ANALYSIS FOR NUCLEAR PIPING SYSTEMS

Luk, Chih-Hung.

Stone & Webster Engineering Corp., Boston Mass.. ASME paper no. 75-PVP-61

Key Words: piping systems, nuclear power plants, method of characteristics, steam hammer

The method of characteristics is applied to steamhammer analysis and nuclear piping system. If the dynamic fluid behavior in the steam chest is not considered, the boundary condition formulated to describe the time-dependent fluid behavior of the steam chest leads to numerical unstable solution. To overcome this difficulty, the dynamic fluid behavior in the steam chest can be described by a single degree mechanical system. The

corresponding flow conditions there are then determined by the time-step amplification method. This dynamic boundary condition reduces the calculated steamhammer loads and helps avoid numerical instability problems in the computing procedure.

76-267

AN ANALYSIS OF WATERHAMMER LOADS IN A TYPICAL NUCLEAR PIPING SYSTEM
Harper, C. R., Hsieh, J. S. and Luk, C. H.
Stone & Webster Engineering Corp., Boston, Mass., ASME paper no. 75-PVP-53

Key Words: water hammer, nuclear power plants, piping systems, computer programs

In the analysis reporte herein, a computer program based on the method of characteristics was used to solve the fluid dynamics equations and obtain waterhammer loads as a function of time. These loads are then used as time-history functions in a piping analysis program which uses modal superposition dynamic analysis to determine deflections, reactions, and stresses.

PLATES AND SHELLS

(Also see No. 265)

76-268

DYNAMIC STABILITY OF AXIALLY-STIFFENED IMPERFECT CYLINDRICAL SHELLS UNDER AXIAL STEP LOADING
Lakshmikantham, C. and Tsui, Tien-Yu
Army Materials and Mechanics Research Center, Watertown Mass., AMMRC-TR-75-10, 11 pp (March 7, 1973)

Key Words: cylindrical shells, dynamic stability, dynamic buckling

The dynamic stability of an axially-stiffened cylinder under an axial impulsive load in the form of a step function is investigated. Donnell-Karman-type nonlinear equations are derived for initially imperfect stiffened cylinders including eccentricity of stiffeners. A buckling criterion is defined, and buckling loads are computed for different imperfection parameters, and positive and negative

eccentricities are computed for a cylinder with an axial stiffening system. The results indicate the high imperfection sensitivity of axially-stiffened cylinders.

AD-A011 895/0GA

76-269

SOUND RADIATION FROM AN ORTHOTROPIC PLATE SUPPORTED BY A DOUBLE SET OF STIFFENERS

Greenspon, J. E.

J G Engineering Research Associates
Baltimore, Md., 0-75-1. 23 pp (June 1975)

Key Words: plates (structural members), sound waves, stiffened plates

A solution is developed for sound radiation from point loading of an infinite orthotropic plate stiffened by two infinite sets of stiffeners. One set is intended to simulate the intermediate or ring stiffeners of a shell; the other set is intended to simulate the bulkheads. Results emphasize the importance of bulkheads at low frequency, the relative lack of importance of rings at low frequency if stiff bulkheads are present, and the relative insensitivity of the sound field to both ring and bulkhead stiffness.
AD-A013 320/7GA

76-270

STABILITY, BEHAVIOUR AND VIBRATIONS OF AN ANNULAR PLATE UNDER UNIFORM COMPRESSION

Technion - Israel Inst. of Tech. Haifa Dept. of Aeronautical Engineering, TAE-229.
68 pp (October 1974)

Key Words: plates, vibration response

The behavior of an annular plate free at its inner edge and simply supported at the outer one is investigated experimentally. The loading is a compressive inplane force which is uniformly applied at the outer boundary. Deformations, strains frequencies and mode shapes are explored over sub, trans and post buckling regions. Experimental results for the buckling loads, obtained by different methods are compared, and comparison is

also made with existing theoretical data. An approximate formula for the vibrations of the loaded annular plates is developed and compared with the experimental results.
AD-A011 487/6GA

76-271

**VIBRATING MODES FOR SIMPLY SUPPORTED
POLAR-ORTHOTROPIC SECTOR PLATES**

Rubin C.

Faculty of Mechanical Engineering, Technion-
Israel Institute of Technology, Haifa, Israel
J. Acoust. Soc. Amer., 58 (4) 841-845
(October 1975) 4 refs

Key Words: natural frequencies, mode shapes,
plates

The method used will provide natural frequencies and mode shapes for the free vibrations of a simply supported polar orthotropic sectorial plate. The special case of a wedge-shaped plate is also treated. The solution applies to any sectorial plate with radial edges simply supported and arbitrary boundary conditions along the circular edges. A series solution that converges rapidly for many natural frequencies is obtained for radial, circular, and mixed modes. The effect of plate orthotropy on the modes is evaluated. The solution is easily reduced to the isotropic case.

76-272

**VIBRATORY BEHAVIOR OF TWISTED
CANTILEVERED PLATES**

MacBain, J. C.

Air Force Aero Propulsion Lab., Wright-
Patterson Air Force Base, Ohio. J. Aircraft
12 (4) 343-349 (April 1975)

Key Words: cantilever plates, finite element
technique, computer programs, vibration
response

A combined numerical-experimental study of the effects of varying tip twist and increasing centrifugal loading on the resonant characteristics of cantilevered plates is presented. NASTRAN is used to compute natural frequencies, mode shapes, and normalized shear

stress distribution for each mode of vibration for cantilevered plates having varying degrees of tip twist and increasing centrifugal loading. For zero centrifugal load, the mode shapes are in good agreement with those obtained experimentally using holographic interferometry. For increasing centrifugal loading, the modal lines of the flexural modes of vibration shift toward the plate root. Increased centrifugal loading also strongly affects the character of some of the plate-like vibration modes of the cantilevered plate.

76-273

**ON THE DYNAMIC BUCKLING OF SHELLS
OF REVOLUTION**

Tong, P. and Adachi, J.

Army Materials and Mechanics Research
Center Watertown, Mass.. AMMRC-TR-75-9
6 pp (October 5, 1973)

Key Words: shells of revolution, finite
element technique, dynamic buckling

Test results of earlier static pressure and pulsed pressure buckling experiments on shallow spherical caps were examined analytically. Imperfection amplitudes were estimated from the static buckling results. The pre-buckling deformation and the bifurcation buckling mode are determined by finite element analysis for the series of spherical caps tested. The present analysis generally gives a lower bound on the experimental data.
AD-A011 300/1GA

76-274

**TRANSMISSIBILITY ACROSS SIMPLY
SUPPORTED THIN PLATES. 1.
RECTANGULAR AND SQUARE PLATES WITH
AND WITHOUT DAMPING LAYERS**

Ochs, J. B. and Snowdon, J. C.

Engineering Acoustics Program, The Penn.
State Univer., Univer. Park, PA 16802.
J. Acoust. Soc. Amer., 58 (4) 832-840
(October 1975)

Key Words: rectangular plates, membranes
(structural members), material damping,
damped structures, transmissivity

The transmissibility across thin, simply supported, rectangular, and square aluminum plates has been determined experimentally. The results were in close agreement with theoretical predictions. The frequency range extended from 25 to 3025 Hz. The rectangular and square plates were uniquely supported by spring-steel flanges designed to simulate the idealized supports assumed as boundary conditions in theoretical analyses. The measured transmissibility across the damped plated could almost be duplicated by an expression for transmissibility developed for an internally damped homogeneous plate having damping factors equal to those of the composite plates.

76-275

THE PENSTOCK VIBRATION CHARACTERISTICS-ON THE VIBRATIONS OF SIMPLY SUPPORTED AND RING-STIFFENED CYLINDRICAL SHELLS FILLED WITH PRESSURIZED WATER

Shiraki, K., Honma, and Nagata, O.
Mitsubishi Heavy Industries, Japan, ASME paper no. 75-DET-114

Key Words: penstocks, cylindrical shells, stiffened shells, fluid-filled containers, vibration response

Penstocks are ring-stiffened, long cylindrical shells filled with pressurized water. A free vibration analysis is constructed for a penstock with simple supported type of end conditions. The present theory has the assumption of zero hoop and shear strain in addition to ones of thin shell theory.

76-276

NONLINEAR VIBRATION OF CYLINDRICAL SHELLS

Chen, J. C. and Babcock, C. D.
Calif. Institute of Tech., Pasadena, Calif.,
AIAA J. 13 (7) 868-876 (July 1975) 11 figs
21 refs

Key Words: nonlinear response, cylindrical shells, perturbation theory

The large amplitude vibration of a thin-walled cylindrical shell was analyzed. A perturbation method was used to solve the steady-state forced vibration problem. The simply-supported boundary conditions and the cir-

cumferential periodicity condition were satisfied. The solution indicated that, in addition to the fundamental mode, the response contained both asymmetric and axisymmetric modes with the frequency twice that of the fundamental mode. Vibrations involving a single driven mode response were investigated: the nonlinearity was either softening or hardening, depending on the mode. The vibrations involving both a driven mode and a companion mode were also investigated. Experimental results were in qualitative agreement with the theory. 'Nonstationary' response was detected at some frequencies for large amplitude response in which the amplitude drifted from one value to another. Various nonlinear phenomena were observed and compared with theoretical results.

76-277

EXTENSIONAL VIBRATION OF THIN PLATES OF VARIOUS SHAPES

Chen, S. S. H. and Liu, T. M.
Arizona State Univer., Tempe AZ 85281
J. Acoust. Soc. Amer., 58 (4) 828-831
(October 1975) 13 refs

Key Words: free vibration, plates

Free extensional (in plane) vibration of thin plates of Hookean material of various shapes was studied. Separate solutions for dilatational and rotational vibrations were obtained. Boundary conditions were satisfied in a least-square sense. Numerical computations were performed for circular, elliptical, triangular, square, and hexagonal plates. The nodal pattern corresponding to each natural frequency for different modes was obtained. Frequency parameters computed for the circular plate were within 0.05% of published results when available; otherwise accuracy was checked by taking additional terms in the series solution and by dividing the boundary perimeter into finer intervals. The circular plate had the lowest fundamental frequency (k_1) at 2.049. The value increases as the shape is changed. The triangular plate has the highest value at 6.733 in dilatational vibration; the frequencies in rotational vibration (h_1) were 3.737 and 16.364, respectively.

76-278
ON PREDICTING THE NATURAL
FREQUENCIES OF SHROUDED BLADED
DISKS

Cottney, D. J. and Ewins, D. J.
Rolls-Royce Ltd., London ASME paper no.
75-DET-113

Key Words: natural frequencies, shrouds,
disks, blades, mathematical models

Means of extending the various detailed and sophisticated mathematical models of complex-shaped components in such a way as to make accurate predictions for a complete blade-disk-shroud assembly are discussed. The results show some serious problems in achieving this aim but provide a useful insight into what component data must be obtained and how its accuracy will affect the final predictions.

RINGS

76-279
INFLUENCE OF BOUNDARY RESTRAINT
AND CURVATURE ON THE VIBRATION OF
CIRCULAR RING SEGMENTS

Rehfield, L. W., Sparrow C. A., and
Evani, S. R. M.
School of Aerospace Engineering, Georgia
Institute of Technology, Atlanta, Georgia.
30332 Israel J. Tech., 13 (1-2) 7-15
(1975)

Key Words: rings, structural elements,
flight vehicles

Local vibration of floating ring frames of fuselages and missile or launch vehicle bodies characterized by motion predominantly between longitudinal stiffeners was studied with the aid of a ring segment structural mode together with appropriate boundary restraint conditions. Idealized restraint conditions corresponding to simple supports and clamping are employed to describe the transverse or bending restraint applied to the ring segments. Idealized, limiting conditions of extensional or tangential restraint at the ring segment ends corresponding to vanishing axial force or vanishing tangential displacement were studied thoroughly in conjunction with both types of bending restraint.

General results, based upon a Donnell-type theoretical analysis are discussed. Representative results are presented for finite extensional end spring stiffness to provide additional insight into the vibration behavior. Sponsor: United States Air Force, Office of Scientific Research

SPRINGS
(Also see No. 223)

STRUCTURAL
(Also see No. 175)

SYSTEMS

AIRCRAFT
(Also see Nos. 162, 168, 172, 180, 183, 184,
185, 187, 194, 195, 198, 201,
202, 203, 204, 221, 241, 244,
186, 313)

76-280
F100 FAN STALL FLUTTER PROBLEM
REVIEW AND SOLUTION
Jeffers, J. D. and Meece, C. E.
Pratt & Whitney Div., United Aircraft Corp.,
West Palm, Fla. J. Aircraft 12 (4) 350-357
21 figs, 6 refs

Key Words: experimental results, airfoils,
fans, aircraft engines, flutter

An airfoil durability problem was investigated experimentally in the first fan rotor of the F100 engine. Laboratory and simulated engine flight tests, an empirical correlation of aeroelastic stability parameters from engine test data, and substantiation testing of the redesign were included in the study. Initial results showed that rotor failure at high-flight Mach numbers and low altitudes was caused by torsional stall flutter instability. Results of empirical correlation indicated that a design free of flutter required a decrease in both normalized incidence and velocity. The correlation also indicated that the flutter was affected by inlet pressure, a heretofore undocumented phenomenon. The results of substantiation testing confirmed that the

redesign made the rotor flutter-free throughout the entire aircraft flight envelope. It was concluded that an improved stall flutter analysis was required to ensure stable fan and compressor rotor designs and that the effect of changes in inlet pressure level on rotor stability was, in part, the result of the accompanying changes in air density and steady-state aerodynamic loading.

76-281

COMPARISON OF THE ACOUSTIC CHARACTERISTICS OF LARGE-SCALE MODELS OF SEVERAL PROPULSIVE-LIFT CONCEPTS

Falarski, M. D., Aiken, T. N., Aoyagi, K., and Koenig, D. G.
NASA Ames Research Center Moffett Field Calif., J. Aircraft 12 (7) 600-604 (July 1975) 8 figs. 10 refs

Key Words: noise generation, test models, aircraft, wind tunnel tests

The acoustic characteristics and the effect of forward speed were determined for four propulsive lift concepts: the over-the-wing externally-blown jet flap (OTW), the under-the-wing externally-blown jet flap (UTW), the internally-blown jet flap (IBF), and the augmentor wing (AW). The data represent the noise generated by the powered-lift system without acoustic treatment, assuming suppression of other noise sources. When scaled to a 45,500 kg aircraft, the OTW concept exhibited the lowest perceived noise levels; the AW was loudest. All configurations emitted noise 10 to 14 PNdb higher than the noise goal of 95 PNdb at 153 m (500 ft). Only the AW has the capability of acoustic suppression to this level. The effect of forward speed did not approach that expected from the relative velocity increments investigated. The dominant low-frequency noise of the OTW and UTW was reduced 2 db by an 80 knot free-stream velocity. The dominant high-frequency noise of the IBF and AW was unaffected by forward speed.

76-282

ACTIVE FLUTTER SUPPRESSION-A FLIGHT TEST DEMONSTRATION

Roger, K. L. and Hodges, G. E.
The Boeing Co., Wichita, Kansas, J. Aircraft 12 (6) 551-556 (June 1975) 19 figs, 5 refs

Key Words: flutter, vibration control, aircraft, experimental data

The Control Configured Vehicles (CCV) B-52 test airplane was twice flown 10 knots faster than its flutter speed; an automatic control system was used for adequate damping. Design, safety considerations, mechanization, ground testing, and flight testing of the flutter mode control system are reported. Comparisons between flight test and theoretical results are presented. The system was tested at heavy and light airplane weights and for compatibility with simultaneous ride control, maneuver load control, fatigue reduction, and augmented stability.

76-283

TRANSONIC STUDY OF ACTIVE FLUTTER SUPPRESSION BASED ON AN ENERGY CONCEPT

Sandford, M. C., Abel, I., and Gray, D. L.
NASA Langley Research Center, Hampton, Va. J. Aircraft, 12 (2) 72-77 (February 1975) 10 figs. 9 refs

Key Words: flutter, vibration control, aircraft

An aerodynamic-energy criterion was used to suppress flutter of a simplified delta-wing model. With both leading- and trailing-edge active controls an increase in dynamic pressure of 22% above the basic wing flutter point was found. With only a trailing-edge active control an increase in dynamic pressure of 30% above the basic wing flutter point at a Mach number of 0.9 occurred. Analytical methods for predicting open- and closed-loop behavior of the model are also discussed.

76-284

**DESIGN AND ANALYSIS OF FLUTTER
SUPPRESSION SYSTEMS THROUGH THE USE
OF ACTIVE CONTROLS**

Steraman, R. O. and Pinnamaneni, R.
Texas Univer. At Austin Dept. of Aerospace
Engineering and Engineering Mechanics
75005 AFOSR-TR-75-0964 210 pp (Jan. 1975)

Key Words: flutter, aircraft vibration,
vibration control, mathematical models

A theoretical study was made to utilize recent advances in structural dynamic analysis, unsteady-aerodynamic theories, control theories, and optimization techniques in the design and analysis of flutter suppression systems. The formulations that appear most promising for design study applications are presented. One class is suitable for a parameter optimization design approach in the frequency domain; another type is suitable to optimal control design techniques.

AD-A012 687/0GA

Sponsor: Grant AF-AFOSR-1998-71

BRIDGES

(Also see Nos. 169, 170, 182, 291)

76-285

**THE TORSIONAL RESPONSE OF A
SUSPENSION BRIDGE STIFFENING TRUSS
MODEL TO TURBULENCE AND TO AN
UPSTREAM OBSTACLE**

Reinhold T. A., Tieleman, H. W., and
Maher, F. J.

Virginia Polytechnic Inst. and State Univer.,
Blacksburg Dept. of Engineering Science and
Mechanics, VPI-E-74-28, 22 pp (Dec. 1974)

Key Words: torsional response, suspension
bridges, model tests, aerodynamic damping,
turbulence

A 1:64 scale section model of a recently constructed suspension bridge was tested in smooth flow and turbulent flow with and without upstream 'obstacle'. Testing was limited to determination of the aerodynamic damping on a model confined to single-degree of freedom torsional motion. The turbulent atmosphere was generated by use of a uniform grid at the test section entrance. The upstream 'obstacle' selected was an almost exact 1:64 replica of a cable-stayed bridge that was basically a flat plate on a narrower box.

BUILDING

(Also see Nos. 169, 170, 176, 207, 209, 215)

76-286

EVALUATION OF EXISTING STRUCTURES

Wiehle, C. K.

Stanford Research Inst. Menlo Park, Calif.

Facilities and Housing Research Dept.

195 pp (Dec. 1974)

Key Words: buildings, blast response, computer
programs, nuclear explosion damage,
reinforced concrete, walls

The objective of the overall research program is to develop an evaluation procedure applicable to existing NSS-type structures and private homes. Past efforts have been concerned with examining exterior walls; window glass; steel frame connections; applications to actual buildings; reinforced concrete floor systems, including restrained slabs; wood-joint floors; and the dynamic inelastic analysis of a steel frame building.

AD-A011 134/4GA

CONSTRUCTION

(Also see Nos. 171, 172)

EARTH

(Also see Nos. 216, 237, 238, 246)

76-287

**DYNAMIC ANALYSIS OF FOOTINGS ON
LAYERED MEDIA**

Kausel, E., Roesset, J. M., and Waas, G.

Struct. Mechanics Sect., Stone & Webster

Engrg. Corp., Boston, MA, ASCE J. Engr.

Mech. Div., 101 (EM 5) 679-693 (October 1975)

6 figs, 18 refs

Key Words: footings, interaction: soil-structure,
finite element technique

A finite element formulation for the dynamic analysis of circular footings resting on or embedded in layered soil strata accurately reproduces the lateral radiation effects through a consistent energy transmitting boundary. Because the boundary can be placed directly at the edge of the footing without loss of accuracy,

storage requirements and computation of time are lower. The analysis must be performed in the frequency domain; arbitrary transient loading conditions are then handled using fast Fourier transformation techniques.

76-288

CYCLIC TRIAXIAL TEST EQUIPMENT TO EVALUATE DYNAMIC PROPERTIES OF FROZEN SOILS

Vinson, T. S.

Michigan State Univer., East Lansing, Dept. of Civil Engineering, MSU-CE-75-1, 43 pp (March 1975)

Key Words: interaction: soil-structure, frozen soils, seismic excitation, earthquakes, test equipment, damping coefficients, shear modulus.

To establish the response interaction between a soil deposit and a structure during an earthquake, two soil properties are required: (1) the dynamic shear modulus, and (2) the damping factor. For unfrozen soils these properties have been determined by several investigators, and design equations and curves to establish the properties for representative soil types have been developed. Equipment to evaluate the shear moduli and damping factors of frozen soils for use in ground response analyses during earthquakes, and develop a research plan to investigate parameters that might influence shear moduli and damping factors of frozen soils are presented.

PB-242 419/0GA

Sponsor: NSF-GK-37439

HELICOPTERS

(See also No. 188)

76-289

HINGELESS ROTOR SERVO-AEROELASTICITY

Young, M. I.

Delaware Univer., Newark Dept. of Mechanical and Aerospace Engineering, ARO-9549.10-E 64 pp (July 24, 1975)

Key Words: flexural vibrations, helicopters, rotary wings

Contents include: Coriolis coupled bending vibrations of hingeless helicopter rotor blades; scale effects in bending vibrations of helicopter rotor blades; influence of pitch and twist on blade vibrations; dynamics of blade pitch control; open and closed loop stability of hingeless rotor helicopter air and ground resonance; stability and control of hingeless rotor helicopter ground resonance; and optimization of the cyclic control response of helicopter rotors. AD-A013 574/9GA

76-290

HELICOPTER TROOP/PASSENGER RESTRAINT SYSTEMS DESIGN CRITERIA EVALUATION

Carr, C. W.

Ultrasystems Inc. Phoenix, Ariz. Dynamic Science Div., USAAMRDL-TR-75-10, 153 pp (June 1975)

Key Words: safety restraint helicopters

A systems analysis for troop restraint systems in helicopters was conducted. Hardware was designed and statically tested in accordance with the requirements of two proposed specifications. Design iterations were required on some hardware components. Three designs were developed for dynamic testing. When testing of the restraint systems revealed additional weaknesses, design modifications were incorporated; testing was repeated until satisfactory results were obtained. Proposed specifications were modified according to test results. The two specifications now define advanced restraint systems providing optimum restraint for occupants of Army aircraft. AD-A012 270/5GA

HUMAN

76-291

HIGHWAY BRIDGE VIBRATION STUDIES

Aramaks, T.

Purdue Univer., Lafayette, Ind. Joint Highway Research Project, JHRP-75-2 275 pp (Feb. 1975)

Key Words: bridges, vibration excitation, human response

The report deals with acceleration studies of highway bridges. The effects of major parameters on the bridge accelerations were investigated and compared to the acceleration criteria for human response. Three different types of highway bridges were investigated: simple span, two span continuous, and three span continuous bridges. The parameters selected included those related to the bridge, to the vehicle and to the initial conditions of bridge and vehicle. The results of the investigation indicated the amplitudes of accelerations which psychologically disturbed the pedestrian. Some magnitudes of acceleration were larger than the recommended limit of comfort when the surface roughness of the bridges was taken into account.
PB-242 557/7GA

76-292

NOISE AND CHILDREN; A REVIEW OF LITERATURE

Mills, J. H.

Dept. of Otolaryngology, Medical Univer. of South Carolina, Charleston, SC 29401, J. Acoust. Soc. Amer., 58 (4) 767-779, (October 1975) 35 refs

Key Words: noise tolerance, human response

A review of information that deals with the effects of noise on children, states potential problem areas, and recommends research.

76-293

VIRTUAL MASS AND DAMPING CORRECTIONS TO A ONE DIMENSIONAL FORMULATION OF COCHLEAR MECHANICS WITH AN APPLICATION TO A THREE CHAMBER MODEL

Chadwick, R. S., Israeli, M., and Levite, U. Technion-Israel Institute of Tech. Haifa, Israel. Israel J. Tech. 13 (3) 168-179 (1975) 12 figs, 7 refs

Key Words: mathematical models, ears

A one-dimensional formulation of cochlear mechanics presented herein, is corrected for fluid motion normal to the vibrating partitions using the concepts of the virtual mass and virtual damping. A three chambered cochlear model including interaction between Reissner's membrane and the basilar membrane is

analyzed. The response of the three chambered model to a pure tone excitation can differ from previous two chamber cochlea models, depending on the stiffness of Reissner's membrane. The accepted place principle of frequency discrimination may have to be modified in light of the results from the present model.

ISOLATION

76-294

MODELING THE IMPACT RESPONSE OF BULK CUSHIONING MATERIALS

McDaniel, D.

Army Missile Research Development and Engineering Lab Redstone Arsenal Ala Aeroballistics Directorate, RD-75-16, 175 pp (May 9, 1975)

Key Words: packing materials, shock absorption, mathematical models, computer programs, viscoelastic properties, polymers, foams

The report deals with the use of bulk cushioning materials in shock mitigation systems. The current techniques used in designing bulk cushioning systems are discussed, and an improved technique is presented through the development of a mathematical model of impact response which is based on the viscoelastic properties of bulk cushioning materials. A General Model of impact response which is applicable to all types of bulk cushioning materials and predicts g-level response in terms of drop height, static stress, thickness of cushion, and temperature is developed. A technique for determining the optimal cushioning system design is developed, and examples of the use of the technique are presented for a cross-linked polyethylene foam cushioning material.

AD-A011 230/0GA

MECHANICAL

(Also see No. 197)

76-295

**RAPID VERIFICATION OF ENGINE ROTOR
AND CASE FLEXIBILITIES BY A MODAL
COMPARISON ALGORITHM**

Marmol, R. A. and Akin, J. T.

Pratt & Whitney Aircraft Div., United Aircraft
Corp., West Palm Beach, Fla., J. Aircraft
12 (4) 242-246 (April 1975) 9 figs. 5 refs

Key Words: mathematical models, rotors,
engines, couplings flexible couplings

A computation technique was developed in which localized dynamic flexibilities in an assembled rotor or case can be rapidly determined from experimental mode shape and frequency data. A dynamic mathematical model of the structure had empirical flexibility terms assigned to mechanical joints. The vibratory response of the structure was measured and compared with calculated values. Agreement between calculated and experimental mode shapes and frequencies was obtained by a computerized random search technique. The technique, developed for rotor critical speed applications, may be applied to any simple or complex beam-type structure.

76-296

NOISE CAUSED BY PISTONS

Steidle, W. and Wacker, E.

Automobiltech 77 (10) 293-298 (Oct. 1975)
10 figs. 8 refs

Key Words: optimization, noise reduction,
pistons, engine noise

Computational methods can be used to optimize piston design with respect to noise, thus minimizing the number of engine tests. Sufficient shank stiffness greatly influences minimization of piston noise. The effects of shape and lubricating conditions in the engine must be investigated. Reduction of overall stress level in the piston shank is possible with constructive techniques. This leads to a reduction in the shank fall-in and to less piston noise, even after a long period of operation. For expansion controlled pistons, larger values of fall-in can be accepted without having more piston noise as in case of pure bimetallic pistons.

76-297

**DRIVE-TRAIN DYNAMICS TECHNOLOGY:
STATE-OF-THE-ART AND DESIGN OF A
TEST FACILITY FOR ADVANCED
DEVELOPMENT**

Badgley, R. H., Smalley, A. J., Fleming, D. P.
and Latham, N. Y.

Mechanical Technology Inc., ASME paper no.
75-DET-74

Key Words: drive trains, test facilities,
reviews

This paper treats the background of flexible dynamic drive-train systems. It discusses the present state-of-the-art in several important areas, and finally describes the design characteristics and capabilities of a drive train test facility which is being constructed for development of advanced drive train dynamics technology and components.

76-298

**DYNAMIC LOAD AMPLIFICATION FACTORS
FOR SAFETY VALVE HEADERS**

Lee, M. Z.

Gilbert Assoc., Inc., Reading, Pa.
ASME paper no. 75-PVP-70

Key Words: valves, dynamic excitation,
dynamic structural analysis

This paper presents a simplified method of calculating the dynamic load amplification factors to be used in the stress analysis of the header under the safety valve discharge loading. The effects of load-time amplification and the valve locations are considered so that dynamic stress can be obtained by employing static analysis.

76-299

**VIBROACOUSTIC PROCESSES IN MACHINES
AND COUPLED DEVICES**

Genkin, M. D.

Joint Publications Research Service, Arlington,
Va., 103 pp (June 4, 1975)

Key Words: machine noise, machinery
vibration, automotive transmissions, gears,
test equipment

The report contains an examination of the problems of vibrations in toothed transmissions and the distribution of the vibrations along the transmission components of the machine and mechanisms.

JPRS-64910

76-300

ASSESSMENT OF POSSIBLE METHODS OF CONTROLLING THE ANOMALOUS VIBRATIONS OF ROTATING WIRE STRANDING MACHINERY

Pierce, A. D.

Georgia Institute of Tech., Atlanta, Ga.

Key Words: vibration control, rigid foundations, machinery

This paper presents guidelines for the analytical aspects of assessing machine designs from the standpoint of predicting the magnitude of and of controlling anomalous vibrations caused by the joint influence of bending stiffness asymmetry and gravity. Particular emphasis is given to the resulting magnitudes of the oscillating forces on the supports, which are shown to vary at low speeds as the square of the rotational speed. Guidelines for designing sufficiently stiff supports to control the vibrations are presented.

METAL WORKING AND FORMING

76-301

DESIGN OF CHATTER-FREE MACHINE TOOLS
Maddux, K., Brown, D., and Schierloh, F.
Structural Dynamics Research Corp., Cincinnati
Cincinnati, Ohio ASME paper no. 75-DET-5

Key Words: computer programs, self-excited vibrations, forced vibrations, machine tools, chatter

This paper describes a convenient, conversational time sharing computer program for the investigation of self-excited and forced vibration of machine tools. The operations of turning, drilling, milling, and grinding can be analyzed.

76-302

MECHANISMS OF AERODYNAMIC NOISE GENERATION IN IDLING WOODWORKING MACHINES

Brooks, T. F. and Bailey, J. R.

NASA Langley Research Center, Hampton, Va
ASME paper no. 75-DET-47

Key Words: woodworking machines, noise generation, aerodynamic characteristics

A combined analytical experimental study is made on the noise due to aerodynamic interaction of rotating cutterheads and stationary table lips in woodworking machines. It applies to all cases of machinery where stationary surfaces are in the immediate vicinity of rotating, cutterheads, e.g., wood planers, jointers, shapers and molders.

PRESSURE VESSELS

76-303

IDENTIFICATION OF THE VIBRATION MODES OF A CAR DRIVEN ON THE ROAD

Talbot, C. R. S., Tidbury, G. H., and Jha, S. K.

Cranfield Institute of Tech., Cranfield,
Bedford, England, ASME Paper No. 75-DET-115

Key Words: natural frequencies, automobiles, modal analysis

A unique method of selecting the modal frequencies of a car from the analysis of the acceleration time histories of points on the structure is described. The shortcomings of selecting modal frequencies from peak amplitude response are discussed. The vibrational modes of the car at some selected frequencies obtained by the above method are presented.

PUMPS, TURBINES, FANS, COMPRESSORS

(Also see No. 243)

76-304¹

AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE DYNAMIC RESPONSE OF A PRESS

Bai, M., and Foster, J. E.
Vibro-Dynamics Corp., La Grange, Ill.,
ASME Paper No. 75-DET-48

Key Words: dynamic response, presses, mathematical models, experimental data

Dynamic simulation studies for the vertical motion of an industrial punch press are described. A mathematical model was generated and solutions were obtained by numerical integration. Field measurements were conducted and compared with computer results to verify the simulation techniques.

76-305

SEISMIC ANALYSIS OF VERTICAL PUMPS ENCLOSED IN LIQUID FILLED CONTAINERS

McDonald, C. K.
Univ. of Alabama in Birmingham, Birmingham, Ala., ASME Paper No. 75-PVP-56

Key Words: pumps, seismic response, fluid-filled containers

A dynamic seismic analysis of vertical pumps which are enclosed in liquid filled cylindrical containers is described. The effect of hydrodynamic coupling between the pump column and container is included.

76-306

A SYSTEMS APPROACH FOR CONTROL OF PUNCH PRESS NOISE

Bailey, J. R., Daggerhart, J. A., and Stewart, N. D.
North Carolina State Univ., Raleigh, N.C.
ASME Paper No. 75-DET-49

Key Words: systems approach, noise reduction, presses

A systems approach developed for control of punch press noise is described. Extensive experimental results are reported to provide examples of the techniques described in the systems approach.

76-307

SUBSTRUCTURES ANALYSIS OF IMPELLER VIBRATION MODES

Mak, S. W., and Botman, M.
United Aircraft of Canada Ltd., Longueuil, Quebec, Canada

Key Words: compressors, natural frequencies, mode shapes, finite element technique

The coupled modes of vibration of curved thin vanes mounted on a solid hub are investigated in this paper by means of Hurty's method of substructures and a finite-element analysis based on Clough's flat triangular element. The results are compared with holographic test data.

76-308**VIBRATION OF A SMALL RECIPROCATING COMPRESSOR**

Ishii, N., Imaichi, K., Karōroku, N., and Imasu, K.

Osaka Electro-Communication University, Neyagawa, Osaka, Japan, ASME Paper No. 75-DET-44

Key Words: reciprocating engines, compressors, equations of motion

The numerical solution of the equations of motion for a compressor system and an experiment with it are described in this paper. The results of noise and vibration studies are discussed.

76-309**VIBRATION OF DERIAZ PUMPS AT DOS AMIGOS PUMPING PLANT**

Ruud, F. O.

U.S. Bureau of Reclamation, Denver, Colo. ASME Paper No. 75-FE-31

Key Words: pumps, shafts, vibration response

This paper described the severe vibration problems encountered during initial operation of a large Deriaz pump. It was found that transient shaft vibrations prevented normal pump shutdown operation when severe counter-rotational whirl occurred as the shaft speed decreased. The vibration at low blade angles precluded operation below half the rated flow and that high intake water levels led to rotational shaft whirl in the normal operating range. Subsequent field testing evaluated various possible sources of vibration excitation, and verified the remedy of air injection into the headcover.

RAIL**76-310****STEADY-STATE VIBRATIONS OF RAIL ON AN ELASTIC DAMPED FOUNDATION SUBJECTED TO AN AXIAL FORCE AND A MOVING LOAD**

Dokainish, M. A., and Elmaraghy, W. H. McMaster Univ., Hamilton, Ontario, Canada ASME Paper No. 75-RT-5

Key Words: periodic response, railroad tracks, elastic foundations, moving loads

The recent practice of continuously welding railroad rails suggests that considerable axial forces may be induced in the rails due to a change in temperature. This paper presents an analytical solution for the effects of an axial force on the steady-state vibrations of a rail continuously supported on an elastic damped (viscoelastic of the Kelvin type) foundation and subjected to a moving load. The presence of damping is shown to result in an unsymmetric dynamic deflection of the rail.

76-311**LATERAL STABILITY OF A SIX-AXLE LOCOMOTIVE**

Garg, V. K., and Mels, K. D.

General Motors Corp., LaGrange, Ill. ASME Paper No. 75-RT-7

Key Words: lateral response, locomotives, mathematical models

Lateral stability of a six-axle locomotive is investigated with a linearized theoretical analysis. Parameters found to have a significant effect on the critical speed for hunting oscillations are: truck wheel base, wheel tread taper angle, the lateral stiffness and amount of damping in the primary suspension, and the yaw moment of inertia of the wheel set and truck frame. Some limitations of the model are pointed out for future investigations.

76-312**DYNAMIC INTERACTIONS OF PRT VEHICLES AND ELEVATED GUIDEWAYS**

Likins, Peter W.

California Univ., Los Angeles, School of Engineering and Applied Science.

UCLA-ENG-7523 DOT/TST-75/104, 51pp (Mar 75)

Key Words: rapid transit railways, guideways, suspension systems (vehicles), mathematical models, equations of motion, interaction: rail-wheel

An analytical basis is provided for preliminary design of Personal Rapid Transit (PRT) system guideways and vehicle suspension systems. Careful attention has been given to the development of high fidelity mathematical models of PRT vehicle dynamics, including primary and secondary suspension systems. The equations of motion of a nine-degree-of-freedom vehicle model traversing a moving guideway are given. The equations of small elastic vibration for both straight and curved elevated guideways, modeled as elastic beams, are included. Vehicle and guideway equations are combined for purposes of system stability analysis and simulation.
PB-243 655/8GA
Sponsor: Contract DOT-OS-40080

REACTORS

(Also see Nos. 211, 212, 227, 266, 267)

76-313

ANALYSIS OF AN IN-PILE REACTOR TUBE
Von Riesemann, W. A., and Gubbels, M. H.
Sandia Laboratories, Albuquerque, N. M.
ASME Paper No. 75-DET-43

Key Words: computer programs, test facilities, nuclear fuel elements

This paper presents the results of an analysis of an in-pile pressure tube and its components which are subjected to a severe dynamic over-pressure pulse. The in-pile tube is part of a closed loop used in the Power Burst Facility at the Idaho National Engineering Laboratory which is used to test power reactor fuels. Hardware, loading conditions and modeling techniques are described. A two-dimensional dynamic nonlinear finite element computer program HONDO was used in the analysis.

RECIPROCATING MACHINES

(Also see No. 308)

ROADS

(Also see Nos. 166, 177, 178)

76-314

MODULAR APPROACH TO STRUCTURAL SIMULATION FOR VEHICLE CRASHWORTHINESS PREDICTION
Pin Tong, and Rossettos, J. N.
Transportation Systems Center, Cambridge, Mass.
DOT-TSC-NHTSA-74-7 DOT-HS-801 475, 37pp (March 1975)

Key Words: crashworthiness, collision research (automotive), mathematical models, finite element technique

A modular formulation for simulation of the structural deformation and deceleration of a vehicle for crashworthiness and collision compatibility is presented. This formulation includes three dimensional beam elements, various spring elements, rigid body elements, and modal elements.
PB-241 784/8GA

76-315

VEHICLE-PAVEMENT INTERACTION STUDY
Olson, R. M., Johnson, J. H., and Gallaway, B. M.
Texas Transportation Inst., College Station
TTI-2-8-69-138-7F, 73 pp (Oct 1974)

Key Words: interaction: wheel-pavement, tires, surface roughness, pavement roughness, experimental data

This final report described methods, procedures, and results of a study made in the laboratory on selected pavement surfaces. Measurements were made on pavement surfaces on Texas highways and on control surfaces constructed at the Texas A and M Research Annex. The effect of rainfall was examined and an equation was developed to relate water depth to other variables. Finally, an expression was developed relating pavement characteristics, water depth, vehicle speed, and tire tread depth to skid or friction number.
PB-242 169/1GA

76-316

**ANALYSIS AND SIMULATION OF VEHICLE/
GUIDEWAY INTERACTIONS WITH APPLICATION TO A TRACKED AIR CUSHION VEHICLE**
Ravera, R. J., and Anderes, J. R.
Mitre Corp., McLean, Va.
MTR-6839 FRA/ORD/D-75-38, 96 pp
(Feb. 1975)

Key Words: tracked vehicles, ground effect machines, interaction: vehicle-guideway, computer programs, digital simulation

Several analytical methods for investigating the problem of vehicle/guideway dynamic interactions are presented. These methods include several digital programs, each tailored to solve a particular aspect of the vehicle/guideway problem. Also included are computerized frequency domain methods for rapid estimation of system sensitivity to principal parameters and for use in selecting candidate guideway parameters. The major tool is the full scale vehicle/guideway dynamic interaction simulation, TRAVSIM, which includes coupled vehicle/guideway dynamics, independently generated guideway roughness profiles, and data processing for obtaining vehicle output data in the various ride quality formats.

PB-242 014/9GA

Sponsor: Contract DOT-FR-30015

76-317

A HUMAN MODEL FOR MEASURING OBJECTIVE RIDE QUALITY
Wambold, J. C., and Park, W. H.
The Pennsylvania State Univ., Univ. Park, Pa.
ASME Paper No. 75-DET-6

Key Words: ride dynamics, anthropomorphic dummies, measurement techniques

This paper describes a system for objective measurement of vehicle ride quality which can be used with all types of transportation systems. The comfort measuring system consists of a ride quality dummy which carries instrumentation to measure the parameters for comfort calculations. Instrumentation and data reduction techniques have been installed to measure comfort in three axes. However, it was found that only the vertical axis presently simulates human response.

76-318

VEHICLE LATERAL DYNAMICS UNDER EXTREME CONDITIONS
Sorgatz, Ulrich, and Ammesdörfer, Fritz
Automobiltech. 77 (4) 124-129, (April 1975), 12 figs, 4 refs

Key Words: mathematical models, ride dynamics, automobile tires

This paper is motivated by the experience that an agreement of simulated and experimental results is difficult to obtain. It is caused by the lack of exact data or adequate modeling assumptions. A method is described, how to combine experiment and simulation to get an agreement of higher order and to extend the knowledge of tyre behavior under extreme slip angle and load conditions. In connection with a general purpose simulation model this is the basis for a reliable prediction of vehicle reactions up to the limits of road holding, cornering ability, or even roll over.

76-319

TESTING OF TYRE UNIFORMITY
Grins, W.
Automobiltech. 79 (2) 46-49, (Feb. 1975)

Key Words: automobile tires, test equipment

A tire testing machine is described in this paper. Within quality control, tires can be checked on force dependent uniformity during continuous running. Also the causes and results of these defects are pointed out. The characteristic feature of this testing machine is a digital computer, which does the data processing mainly and a part of the control.

76-320

FREQUENCY RESPONSE OF TYRES
Weber, Rudiger, and Persch, Hans-Georg
Automobiltech. 77 (2) 40-46, (Feb. 1975)
12 figs, 35 refs

Key Words: automobile tires, ride dynamics, frequency response

Frequency response of tyres must be taken into consideration if theoretical studies on vehicle dynamics are to be realistic. It has been proven that steady state and transient tyre properties are different from each other. The effect of tire frequency response on vehicle dynamics results is examined in this paper.

76-321

FORCES AND RELATIVE MOTIONS IN THE CONTACT AREA OF STRAIGHT-LINE ROLLING TYRES

Gerresheim, Manfred, and Hussmann, Albrecht W.

Automobiltech. 77 (6) 165-169 (June 1975) 13 figs, 2 refs

Key Words: automobile tires, ride dynamics

Forces and relative motions in the contact area of a conventional tire have been measured for various speeds and several braking and driving slip values. The most important results are shown in 13 graphs. These graphs contain information on how improvements of the friction coefficients of tires can be obtained.

76-322

IMPROVING A MATHEMATICAL MODEL OF AN AUTOMOBILE SEAT WITH RESPECT TO SEAT UPHOLSTERY FRICTION

Anselm, Dieter

Automobiletech. 75 (6) 180-185, (June 1975) 13 figs, 5 refs

Key Words: collision research (automotive), mathematical models, automobile seats

This report contains an improved mathematical model of an automobile seat from the aspect of spring characteristics and friction. Computer models were compared.

76-323

THE INFLUENCE OF DIFFERENT BRAKING DECELERATION ON THE DRIVING CHARACTERISTICS DURING CORNERING

Gauß, Fritz, and Rompe, Klaus

Automobiltech. 77 (7/8) 207-212, (July/Aug. 1975) 14 figs, 13 refs

Key Words: ride dynamics, automobile tires, braking effects

Using a complex mathematical model, the characteristics of two different vehicles with fixed controlled steering wheel during cornering and braking were examined.

76-324

AN OPTICAL CORRELATION METHOD FOR THE DIRECT MEASUREMENT OF TRANSIENT SIDESLIP AND SLIP ANGLES OF MOTOR VEHICLES

Zomotor, Adam

Automobiltech. 77 (7/8), (July/Aug. 1975), 13 figs, 13 refs

Key Words: slip angle, correlation technique, testor vehicles, ride dynamics

This paper presents a method for the direct measurement of sideslip and slip angles. The method is based on the principle of the non-contact velocity measurement at the vehicle in two coordinates. From the two measured velocity vectors, the instantaneous guide motion and thus the sideslip angles, respectively the slip angles can be determined.

ROTORS

76-325

GENERALIZED DYNAMIC SIMULATION OF SKID IN RAIL BEARINGS

Gupta, Pradeep K.

Mechanical Technology Inc., Latham, N.Y.

J. Aircraft, 12 (4) 260-265 (April 1975)

8 figs, 19 refs

Key Words: ball bearings, skid resistance, wear, computer programs

Skid in ball bearings is determined in terms of a generalized integration of the differential equations of motions of the ball under prescribed traction-slip relationship at the ball race contacts. A traction-slip relationship was postulated, and the motion of the ball was examined when one race was subjected to an angular acceleration. The solutions predicted both the magnitude of slip velocities and the expected wear rates for a prescribed wear coefficient. Influence of preload was examined, and the method could be used to determine the required preload to prevent skidding. A computer program was able to predict skid and ball motion as a whole in angular contact ball bearings.

76-326

HIGH-SPEED ROTOR DYNAMICS--AN ASSESSMENT OF CURRENT TECHNOLOGY FOR SMALL TURBOSHAFT ENGINES
Vance, John M., and Royal, Allen C.
Univer. of Florida, Gainesville, Fla.
J. Aircraft, 12 (4) 295-305, (April 1975)
10 figs, 61 refs

Key Words: bearings, rotor response, turbo machinery, helicopter engines, aircraft engines

Research in rotor dynamics needed to solve problems encountered in small high-speed turboshaft engines for helicopter and aircraft propulsion was studied. The philosophy of rotor-bearing system design and methods for critical speed prediction and high-speed balancing were reviewed. The trend to higher speeds requires balancing through flexural modes. The major parameters are bearing support properties. Nonsynchronous excitation was categorized according to the mechanisms producing the forces; better methods are needed to identify the resulting whirling and vibration. The potential for special applications of oil-film and gas bearings was discussed.

76-327

DYNAMIC RESPONSE OF VISCOUS-DAMPED MULTI-SHAFT JET ENGINES
Hibner, David H.
Pratt & Whitney Aircraft Div., United Aircraft Corp., East Hartford, Conn.
J. Aircraft, 12 (4) 305-312, (April 1975)
9 figs, 12 refs

Key Words: dynamic response, viscous damping, jet engines

This paper presents an efficient analytical technique capable of predicting the vibratory response of an engine with nonlinear viscous damping. A unique transfer-matrix method applied to the idealized equivalent engine system produces an unusually small array of influence coefficients. The damper equations for a closed-end viscous damper are derived from the basic Reynolds equation. The analysis is applied to a two-shaft aircraft engine to illustrate the basic concepts of multi-shaft critical speeds and nonlinear viscous-damped response.

76-328

PHILOSOPHY, DESIGN, AND EVALUATION OF SOFT-MOUNTED ENGINE ROTOR SYSTEMS
Magge, Natesh
General Electric Co., Lynn, Mass
J. Aircraft, 12 (4) 318-324, (April 1975)
12 figs, 9 refs

Key Words: computer program, engine mounts, aircraft engines, bearings

The design philosophy, criteria, and methods of evaluation for soft-mounted turbine engine rotor systems used in General Electric aircraft engine design are described. A computer program for system vibration and static analysis (VAST) is capable of finding natural frequencies, normalized modes, and responses due to any distribution of exciting forces considering gyroscopic and shear-deflection effects. Aircraft mounting and excitations from the helicopter rotor are also included in the computer analysis. General Electric's T700 turboshaft engine, under development for the U.S. Army, illustrates

the squeeze film, soft-mounting concept of design. Results from tests of the T700 engine, Advanced Technology Axial Centrifugal Compressor (ATACC), T64 turboshaft, TF34 turbofan, and other engines verify the advantages of soft-mounted rotor systems.

SHIP

(Also see No. 232)

76-329

TORSIONAL VIBRATIONS IN MARINE INSTALLATIONS

Geislinger, Leonhard

MTZ Motortech. 3. 36 (2) 51-53 (Feb. 1975)
2 figs, 1 ref

Key Words: torsional vibration, marine engines, vibration control

Marine installations using Diesel engines and reduction gears usually have a flexible coupling between the engine and the reduction gear; these plants are more complex with regard to torsional vibrations than direct drive installations. It is important to size all parts carefully with regard to torsionals, especially if several engines are used to drive reduction gears. Calculations should not be limited to resonances, and measurements should be made at several points. Dangerous torsional vibrations can exist in gears if ways of avoiding such situations are indicated. Similar situations arise if alternators or generators with high speeds are driven by slow speed main-engines.

SPACECRAFT

(Also see Nos. 161, 193)

76-330

AEROLOADING AND DYNAMICS OF A CONICAL VEHICLE DURING PENETRATION OF A BLAST WAVE

Dunn, J. R., Barbin, A. R., and Vachon, R. I.

Auburn Univ. Ala. Thermoscience Group
26 pp (April 1974)

Key Words: shock waves, missiles, transient excitation

The report deals with the procedure for predicting transient loads on a conical vehicle during the oblique penetration of a blast induced shock wave. The importance of this analysis is in the application of the predicted aerodynamic loads to estimate the dynamic response of the missile during the blast penetration transient.

AD-A011 288/8GA

76-331

VIBRATION AND FLUTTER ANALYSIS OF REUSABLE SURFACE INSULATION PANELS

Dowell, Earl H.

Princeton Univ., Princeton, N.J.

J. Spacecraft and Rockets, 12 (1) 44-50
(Jan. 1975) 12 figs, 5 refs

Key Words: aerodynamic stability, spacecraft components, flutter, vibration response, panels

The Thermal Protection System (TPS) heat shield panels of the space shuttle (and other high performance vehicles) consists of a relatively thick ceramic tile mounted on a soft (viscoelastic) foundation, a so-called "strain-isolator," which in turn is bonded to the primary load carrying metal structure. These are referred to as Reusable Surface Insulation (RSI) panels. The aeroelastic behavior of such panels is reported, principally the stability or "flutter" behavior; much of the work is also relevant to the ("acoustic") response problem.

STRUCTURAL

(Also see No. 160)

76-332

**THE RELATION BETWEEN PICKING NOISE
AND COMPONENT VIBRATIONS IN AUTOMATIC
TEXTILE LOOMS**

Johnson, G. E., and Pierce, A. D.
Tennessee Eastman Co., Kingsport, Tenn.
ASME Paper No. 75-DET-45

Key Words: textile looms, noise measure-
ment, vibration measurement

In this paper experiments are described in which octave band filtered accelerations were measured at 39 points on a fly shuttle type loom. The results of simultaneous sound pressure level measurements are given. A very simple approximate model believed to be applicable for higher frequency bands is described.

TURBOMACHINERY

(Also see Nos. 220, 326)

USEFUL APPLICATION

(Also see No. 242)

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EDITORS RATTLE SPACE

THE GOOD VS THE NOT SO GOOD

In recent conversations with fellow engineers I have heard diverse opinions about the current state of the engineering employment market: some think that demand is soft -- others say that it is adequate. I feel that good engineers are in demand; which brings up a point: what is the difference between a good engineer and a not-so-good one?

In my opinion, a good engineer is adequately equipped with the "tools of the trade" -- that is, he is either knowledgeable regarding the techniques, procedures, and theories necessary to do a specific job or he is willing to learn within a short time how to do the job expected of him. Regardless of the job --- whether it be testing, planning, design, or analysis -- the good engineer should keep himself abreast of the latest technology and be able to apply it profitably.

A good engineer should also be aware of time and costs. On the one hand are the objectives of management to cut costs and time; on the other hand is the ability of the good engineer to prepare realistic schedules, keep management informed of job progress, and to meet deadlines. A good engineer does the job that needs to be done and no more; in other words, he doesn't "work a problem to death." In my opinion this tendency to overdo a given job is one of the major failings of engineers. I believe that awareness of recent developments in his field and of when a problem has been adequately solved are two of the most important aspects of a good engineer.

Of course, it is the responsibility of management to provide the necessary opportunities for engineers to do a good job. Thus management demands with regard to energy expenditure and work load must be balanced with the engineer's need to take time to "catch up" with his field. Lack of opportunity for the engineer to participate in pertinent educational conferences and short courses hinder him and eventually result in a not-so-good engineer.

In conclusion, I believe that adequate career opportunities exist for well-trained and efficient engineers who demonstrate their worth and are given the time necessary to maintain their skills, even if it requires that they come to an "understanding" with management regarding time allotments.

R.L.E.

LITERATURE REVIEW

survey and analysis
of the Shock and
Vibration literature

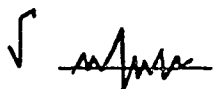
The monthly Literature Review, a subjective critique and summary of the literature, consists of two to four review articles each month, 3,000 to 4,000 words in length. The purpose of this section is to present a "digest" of literature over a period of three years. Planned by the Technical Editor, this section provides the DIGEST reader with up-to-date insights into current technology in more than 150 topic areas. Review articles include technical information from articles, reports, and unpublished proceedings. Each article also contains a minor tutorial of the technical area under discussion, a survey and evaluation of the new literature, and recommendations. Review articles are written by experts in the shock and vibration field.

In this issue, the literature is reviewed on the vibration of rotating machines as they pass through critical speeds and on methods for the analysis of structural frequency-response measurement data.

The vibration of rotors, in the machine critical speed zone, is mathematically characterized by Dr. Iwatsubo of the Kobe University. A through review of the literature involving these techniques is contained in the article.

An introduction to the graphical analysis of frequency-response measurement data is contained in an article by Dr. Radeş of the Polytechnic Institute Bucharest. His article deals with grounded and ungrounded linear systems as well as nonlinear systems.

METHODS FOR THE ANALYSIS OF STRUCTURAL FREQUENCY-RESPONSE MEASUREMENT DATA



MIRCEA RAȘCU*



The purpose of this review is to provide an introduction to the graphical analysis of frequency-response measurement data. The most frequently used data-reduction techniques are reviewed, particularly the analysis of vector diagrams. Accuracy with regard to natural frequencies close to each other, nonlinearities, and imperfect mode isolation are also considered.

Frequency-response analysis plays an important role in engineering (29, 55, 67). For example, structural frequency-response data are used to determine natural frequencies and mode shapes and to identify such system parameters as damping, dynamic stiffness, and generalized masses. These parameters are often helpful in predicting responses to various excitations or in improving dynamic behavior by design modifications (preferably using a mathematical model of the structure). Calculation of the response of a complex structure from frequency-response data obtained for subsystems is beyond the scope of this article.

Reliable results for a specific problem are obtained when appropriate experimental techniques and data-reduction techniques are chosen. Available methods differ with respect to excitation procedures used, physical quantities measured, and the ways in which these are analyzed (7). Single-point excitation followed by graphical data-reduction is most frequently used in industrial resonance tests (75). Methods that combine experimental techniques with the digital computer have been developed especially for the modal analysis of aerospace structures (26, 53, 64, 66). When using these methods, it is usually assumed that the structure is linear and has time-invariant parameters; that the generalized damping matrix related to the associated conservative system is diagonal; and that the force magnitudes are independent of frequency. Multi-point excitation techniques

are used in problems in which pure mode excitation is sought. Papers on this subject are concerned more with techniques for determination of either the excitation forces (4, 17, 18, 56, 94, 98, 105) or the structure (106) and thus will not be discussed below.

Vibration measurements under normal operating or environmental conditions are carried out by either using correlation methods (23, 68, 96) or by simply varying the running speed of the machine and determining resonance (86).

Classification of Techniques.

According to the type of forcing function used, techniques for structural frequency-response measurements, are usually classified as follows: (i) steady-state harmonic, (ii) quasi steady-state, (iii) transient, and (iv) continuous random. If the system is linear and its parameters do not vary with time, the frequency-response measurement is independent of the testing technique used. Steady-state (constant frequency) methods have been widely used in the past (5, 32, 48, 71-73, 108) and are still considered the most accurate for some purposes (51, 53, 63, 91). The bibliography by Ewins (25) enumerates many of the major contributions to this problem. The procedure--exciting the structure harmonically and measuring the response at discrete frequencies--is time-consuming, however, and inconvenient for testing structures having time-variable ambient conditions. Quasi steady-state (slow frequency sweep) methods are popular at present because transfer function analysis equipment is available commercially (50). The forcing frequency of a sine input is swept through the frequency range of interest at a sufficiently slow rate to develop response at all levels of vibration (40). This requirement is most stringent for lightly-damped structures (82).

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Transient methods are now being used to reduce test time; the complex frequency-response function is calculated from the Fourier transform of the excitation and response time histories (111). Digital computers and real time analyzers permit on-line analyses of the structural response. Pulse-type forcing functions have been described (10, 76, 88, 114), but, in these cases, spectra fall rapidly with increasing frequency and exhibit zero values at certain frequencies. Rapid sweep sinewaves have been analyzed (82, 90, 111). Techniques for automatic generation of such input signals, whose spectra have constant mean values over a large frequency range, have been described in detail (109-112). The effects of noise (45), nonlinearities (115), analog to digital conversion (41), and excitation method used (41, 42) have also been taken into account. Alternative methods that utilize decaying vibrations after harmonic excitation has been interrupted have been discussed (19, 107). Finally, random loading methods, in which the input signal is applied by an external source, have been presented (12, 16, 37, 44, 47, 54).

Frequency-response data are either stored for subsequent (analog or digital) processing or displayed (recorded) for graphical analysis in one of the following forms: (i) as plots of modulus against frequency and phase angle against frequency; (ii) as plots of the real (in-phase) and the imaginary (in-quadrature) components versus frequency; and (iii) as a vector diagram of the real component plotted against the imaginary component. The representation of vibration response by vector components was suggested (1, 9, 99) and then developed (14, 48). This subject will be considered in greater detail below.

Methods for fitting an algebraic expression of a transfer function to an experimentally measured frequency response have been dealt with (39, 43, 49-52, 81, 89). This problem will be presented in another review paper.

Grounded Systems.

During normal mode analysis of grounded mechanical systems having no resonances close to each other, it is assumed that any frequency-response function can be represented by summing the response of a number of single-degree-of-freedom systems, each

having a certain modal contribution. Thus, the lumped parameter model (Fig. 1) will be used as a reference in the following discussion of current techniques for the analysis of frequency-response plots.

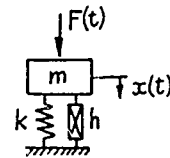


Fig. 1

For harmonic force excitation of the form $F(t) = F_0 e^{i\omega t}$, the governing equation of motion is

$$m\ddot{x} + (h/\omega)x + kx = F_0 e^{i\omega t}, \quad (1)$$

in which m , k , and h are respectively, the mass, stiffness, and coefficient of equivalent hysteretic damping (60).

For a steady-state solution of the form $x = X^* e^{i\omega t}$, equation (1) yields the displacement complex amplitude

$$X^* = \frac{F_0}{k} \frac{1}{\left\{ 1 - \left(\frac{\omega}{\omega_n} \right)^2 + ig \right\}}, \quad (2)$$

in which

$$g = \frac{h}{k} \quad \text{and} \quad \omega_n = \frac{k}{m}. \quad (3)$$

This can also be written

$$X^* = X_e e^{i\phi} = X_R + iX_I, \quad (4)$$

where X is the modulus; ϕ , the phase angle; X_R , the real (in-phase) component; and X_I , the imaginary (in-quadrature) component.

Peak-amplitude method.

The peak-amplitude method is based on Figure 2, which is a plot of the modulus X versus frequency. Amplitude resonance is defined as occurring at point M, where the response reaches a peak. For $\frac{dX}{d\omega} = 0$ one obtains $\omega_{res} = \omega_n$ and $X_{res} = \frac{1}{g} \frac{F_0}{k}$.

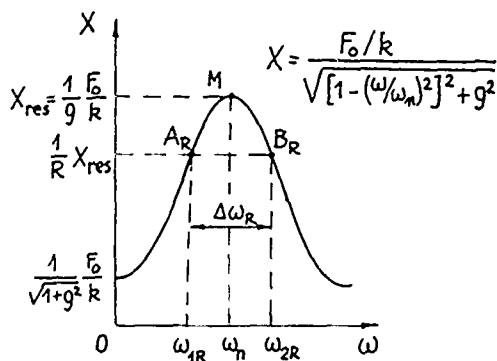


Fig. 2

If ω_{1R} and ω_{2R} are the frequencies at which the response amplitude is $\frac{1}{R} X_{res}$, the damping factor is given by

$$g = \frac{\omega_{2R}^2 - \omega_{1R}^2}{2 \omega_n^2 R^2 - 1} \quad (5)$$

which, for lightly-damped systems, becomes

$$g = \frac{\omega_{2R} - \omega_{1R}}{\omega_n R^2} \quad (6)$$

For $R=2$, A_R and B_R are the "half-power points" of frequencies $\omega_{1,2} = \omega_n \sqrt{1 \pm g}$ and the familiar equation (14)

$$g = \frac{\omega_{2R}^2 - \omega_{1R}^2}{2 \omega_n^2} = \frac{\omega_{2R}^2 - \omega_{1R}^2}{\omega_{2R}^2 + \omega_{1R}^2} \quad (7)$$

is obtained; for negligible damping this equation yields

$$g = \frac{\omega_{2R} - \omega_{1R}}{\omega_n} \quad (8)$$

The dynamic stiffness can be calculated from the value of the displacement amplitude at resonance:

$$k = \frac{1}{g} \frac{F_0}{X_{res}} \quad (9)$$

and the mass from equation (3).

The peak-amplitude method is limited to lightly-damped systems with no natural frequencies close to each other; otherwise, the

peaks are not clearly defined (7, 83). A refined method for evaluating damping solely from the amplitude-frequency curve has been presented (35); in this case, it is assumed that the contribution of the off-resonant modes of vibration is constant in the neighborhood of resonance.

For mechanical impedance measurements (61) the resonance curve of Figure 2 is plotted using "log-log" coordinates. The mass and stiffness components of the single-degree-of-freedom system are deduced from the asymptotic behavior of its response away from the resonance region. The analysis of such impedance-frequency graphs has been extensively developed by Salter (85); he uses a "skeleton-method" to formulate the mathematical model of the structure being studied.

An alternative method (32, 36, 77) is based on the analysis of the diagram of $F_0/(X k)$ versus ω . Measurements are taken at constant displacement amplitude ($x = \text{const.}$), and the

force variation is plotted against frequency. Resonance is defined as occurring at the frequency ω_n at which the force reaches a minimum. The damping factor is given by equation (5), where ω_{1R} and ω_{2R} are the frequencies at which the force is R times the force at resonance. Critical discussions of peak-amplitude methods have been published (71, 91, 97).

Phase-Angle Method.

Variation of the phase angle with frequency is shown in Figure 3. "Phase resonance" occurs at the frequency ω_n where $\phi = -90^\circ$. The half-power points A and B, defined by frequencies ω_1 and ω_2 , correspond to phase angles of -45° and -135° respectively; thus, the damping factor can be evaluated using equation (7).

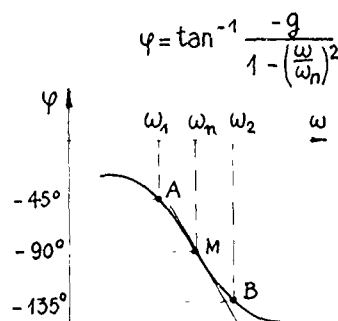


Fig. 3

For many-degree-of-freedom systems it is better to locate the resonance located at the point at which the phase angle curve has an inflection point, $\frac{d^2\phi}{d\omega^2}=0$; the damping factor can then be calculated from the slope of the tangent to the curve at that point (71):

$$g = - \frac{2}{\omega_n \left(\frac{d\phi}{d\omega} \right)_{\omega=\omega_n}} \quad (10)$$

In-Phase Component Method.

Figure 4 shows the variation of $X_R = X \cos \phi$ with ω . Resonance is located at point M, where $X_R=0$ and $\frac{dX_R}{d\omega}$ is a maximum. The half-power points A and B correspond to $|X_R|_{\max}$ so that equation (7) can again be used for evaluating the damping factor.

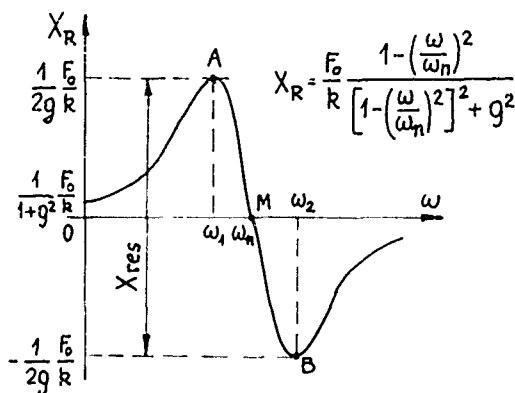


Fig. 4

The best results are sometimes obtained by locating the resonance at the inflection point, where $\frac{d^2X_R}{d\omega^2} = 0$, and calculating the damping factor from the slope of the tangent at the point of inflection (63, 65, 87).

$$g = \frac{2 X_{\text{res}}}{\omega_n \left(- \frac{dX_R}{d\omega} \right)_{\omega=\omega_n}} \quad (11)$$

where X_{res} is measured as the distance between the tangents at points A and B.

In-Quadrature Component Method.

Figure 5 shows the diagram of $X_I = X \sin \phi$ versus ω . Resonance is located at the point where X_I has a peak, $\frac{dX_I}{d\omega} = 0$. It can be seen that X_I peaks more sharply than the total response X and that resonance is equal to the total response X_{res} since the in-phase response is zero.

The half-power points correspond to frequencies at which the quadrature response has half the maximum amplitude; thus, equation (7) can once again be used for evaluating the damping factor. If the frequencies of the points at which the amplitude of X_I is $\frac{1}{\sqrt{2}} X_{I \max}$ are used a formula similar to equation (5) can be obtained. Methods using the diagrams of both in-phase and quadrature components have been described (32, 63, 92).

Using the complex power applied to the structure (11) results in similar diagrams and, in addition, has the advantage of being independent of the damping model.

Vector Diagram Method.

By eliminating ω between the expressions of X_R and X_I , a circle of equation

$$\left\{ X_I + \frac{1}{2g} \frac{F_0}{k} \right\}^2 + X_R^2 = \left\{ \frac{1}{2g} \frac{F_0}{k} \right\}^2 \quad (12)$$

is obtained (Fig. 6), which is the locus of the response vector in the complex plane. It can be seen that the resonance of both amplitude and phase criteria (also for $X_R=0$, $\frac{d\phi}{d\omega} \max$, $\frac{dX_R}{d\omega} = 0$) occurs at point M on the imaginary axis, where $\omega = \omega_n$. The damping factor can be determined from the half-power points, equation (7), which are the ends of the diameter AB perpendicular to OM.

Polar diagrams of systems with many degrees of freedom are not circles, but curves with many loops, usually one for each resonance. The data reduction technique of Kennedy and Pancu (48) has the best capability for separating closely-spaced resonance frequencies. The following must be assumed: (i) lightly-damped linear systems, (ii) constant (invariant with respect to frequency) contribution from the off-resonance modes of vibration, and (iii) no damping coupling between the normal modes. If the system is excited by a harmonic force

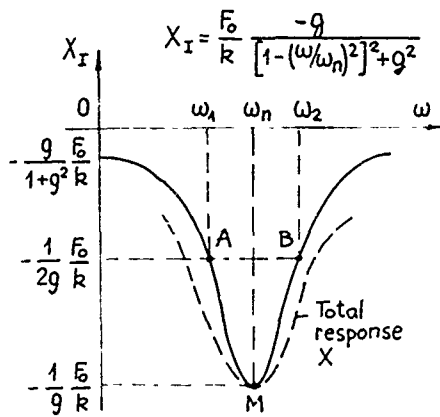


Fig. 5

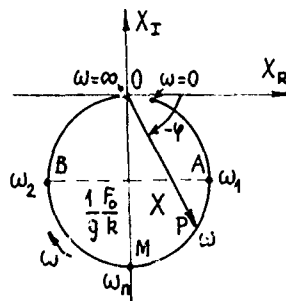


Fig. 6

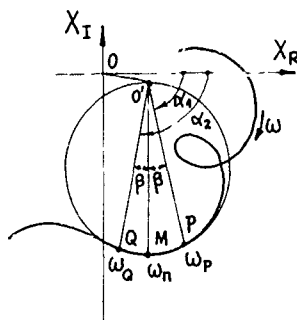


Fig. 7

of constant amplitude and if the displacement response is plotted point by point at equal frequency increments $\Delta\omega$, the distance between two successive points is a maximum at resonance. This is based on the fact that the quantity

$$\frac{ds}{d\left(\frac{\omega^2}{\omega_n^2}\right)} = -\frac{1}{g} \frac{d\phi}{d\left(\frac{\omega^2}{\omega_n^2}\right)} = \chi^2$$

is maximum at $\omega = \omega_n$, where ds is the arc length corresponding to a variation $d\phi$ of the phase angle. The "best circle" is then fitted and drawn through the points in the neighborhood of each resonance (Fig. 7). The diameter $O'M$ of such a circle gives the response at resonance in only one mode of vibration only: the mode that is used to determine the mode shapes (73, 108).

If the half-power points cannot be located on the response diagram, the damping factor can be evaluated from the following equation (60):

$$g = \frac{\omega_q^2 - \omega_p^2}{\omega_n^2} \frac{1}{\alpha_2 - \alpha_1}, \quad (13)$$

where ω_q and ω_p are the frequencies of two points Q and P in the vicinity of resonance and α_2 and α_1 are the angles of the corresponding vector radii with the positive real axis.

$$g = \frac{\omega_q^2 - \omega_p^2}{2\omega_n^2} \cot \beta \quad (14)$$

and, again, measurements can be carried out at almost constant amplitude. This is particularly useful when small nonlinearities are present.

With reference to Figure 8 and based on equation (15), found in (69)

$$g = \left| 1 - \left(\frac{\omega}{\omega_n}\right)^2 \right| \cot \gamma, \quad (15)$$

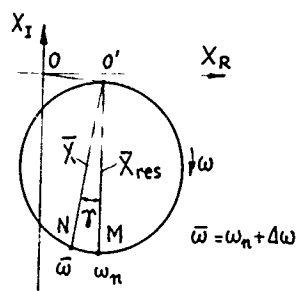


Fig. 8

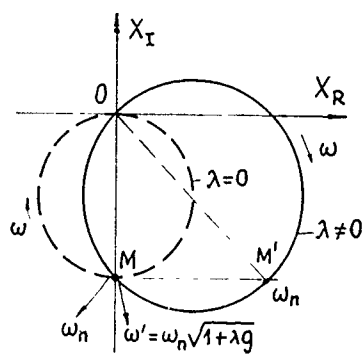


Fig. 9

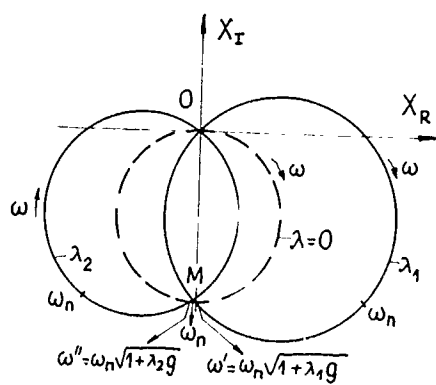


Fig. 10

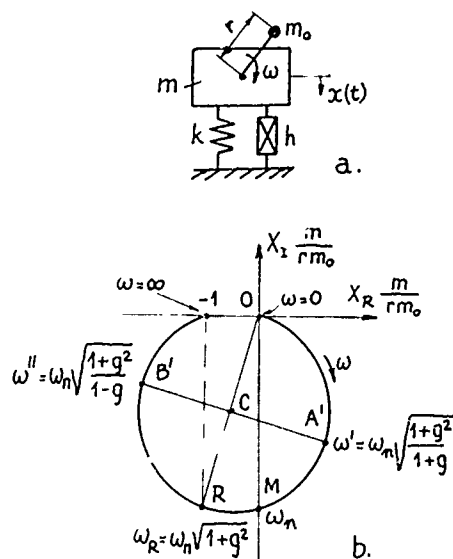


Fig. 11

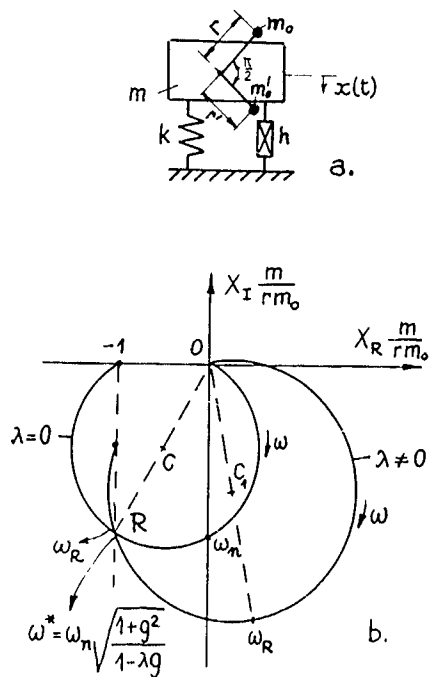


Fig. 12

alternative formulas for the damping factor have been used by Natke (63, 65)

$$g = \left| 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right| \left\{ \left| \frac{\lambda}{\lambda_{res}} \right|^2 - 1 \right\}^{-\frac{1}{2}} \quad (16)$$

by Wolfe and Kirkby (117) for light damping

$$g \approx \frac{\gamma \omega}{\omega_n} \left\{ 2 + \frac{\gamma \omega}{\omega_n} \right\} \cot \gamma, \quad (17)$$

and by Kennedy and Pancu (48) also for light damping

$$g \approx 2 \frac{\lambda}{\lambda_{res}} \left| \frac{\omega}{\omega_n} - 1 \right| \left\{ 1 - \left| \frac{\lambda}{\lambda_{res}} \right|^2 \right\}^{-\frac{1}{2}} \quad (18)$$

Analysis of vector diagrams of frequency-response functions has been extensively used. For a single-degree-of-freedom system with viscous damping, the vector diagram of the displacement is not a circle (8); therefore, the Kennedy and Pancu method applies only for light damping (86). Woodcock (118) extended the study to systems with many degrees of freedom. But in the case of viscous damping, the vector diagram of the velocity is a circle (27, 102, 104) with the diameter on the real axis; the analysis is based on formulas somewhat similar to the above (77).

Methods for fitting the best circle to the experimental points have been presented (21, 86, 103). The consequences of imperfect mode isolation on vector diagrams have been discussed by Gauzy (33, 34) and Pendered and Bishop (71-73). Aspects of modal shape determination can be found (22, 30, 73, 92, 108). Systems with closely spaced natural frequencies have been considered (3, 20, 74, 95, 97, 110). A graphical method for optimization of vibration absorbers using vector diagrams has been given (93). Dynamic acceptance criteria for machine tools have also been proposed (84) using the graphical analysis of vector diagrams.

Comparative critical evaluations of all of the above methods have been presented (7, 15, 58, 63, 71). All of the methods are based on simplification of assumptions concerning damping (2, 76). New methods that take into consideration the complex natural modes of

damped structures have been presented by Klosterman (51, 52), Kussner (53), Natke (64, 66), and Wittmeyer (116).

Method of Forces in Quadrature.

The method of forces in quadrature is another technique of interest in analyzing structural frequency response; it was proposed by de Vries (194) as a variant of the displaced frequencies method (33). The basic approach is to excite the structure in the conventional manner and then plot the vector diagram of the response displacement to a force of constant amplitude $F' = F_0$. A second component of amplitude $F'' = \lambda F_0$ is then added to the primary force and shifted by $\frac{\pi}{2}$, so that the total force becomes $F(t) = F' + iF'' = (1 + i\lambda) F_0 e^{i\omega t}$. The vector diagram of the response to this force is a circle that crosses the basic circle at the resonance point M (Fig. 9). The damping factor is given (for $\lambda > 0$) by

$$g = \frac{1}{\lambda} \left\{ \frac{\omega'^2}{\omega_n^2} - 1 \right\}, \quad (19)$$

where ω_n and ω' are the frequencies, respectively, of the point M on the circles $\lambda = 0$, and $\lambda > 0$.

If two circles of parameters λ_1 and λ_2 are plotted together with the circle $\lambda = 0$ (Fig. 10), both cross the primary circle at the resonance point M; the damping factor can then be calculated using the following equation (62)

$$g = \frac{1}{\lambda_1 - \lambda_2} \left(\frac{\omega'^2}{\omega_n^2} - \frac{\omega''^2}{\omega_n^2} \right) = \frac{\omega'^2 - \omega''^2}{\lambda_1 \omega''^2 - \lambda_2 \omega'^2} \quad (20)$$

where ω_n , ω' and ω'' are the frequencies of the point M on the three circles $\lambda = 0$, λ_1 and λ_2 respectively. When $\lambda_1 = +1$ and $\lambda_2 = -1$, then $\omega' = \omega_2$, $\omega'' = \omega_1$ and equation (20) becomes equation (7). Measurements are carried out at constant displacement amplitude--which is useful for nonlinear systems--and the resonance location is independent of the coordinate system. Close natural frequencies can also

be located easily and accurately.

In a further refinement of these methods (77, 80), constant-frequency lines have been used to locate resonance on the primary circle. Use of additional masses in the method of displaced frequencies has been discussed (28, 33, 46, 59, 100).

Rotating Unbalance.

In rotating unbalance the excitation force is of the form $F(t) = m_0 r \omega^2 e^{i\omega t}$ (Fig. 11 a), and the vertical displacement of the mass m is given by

$$x^* = \frac{r m_0}{m} \frac{\left\{ \frac{\omega}{\omega_n} \right\}^2}{1 - \left\{ \frac{\omega}{\omega_n} \right\}^2 + 1g} \quad (21)$$

The vector diagram (Fig. 11 b) is a circle of equation

$$\left\{ x_r \frac{m}{r m_0} + \frac{1}{2} \right\}^2 + \left\{ x_l \frac{m}{r m_0} + \frac{1}{2g} \right\}^2 = \frac{1 + g^2}{4g} \quad (22)$$

Amplitude resonance occurs at point R where $\omega_r = \omega_n \sqrt{1+g^2}$. Phase resonance ($\phi = -90^\circ$) occurs at point M where $\omega = \omega_n$. The damping factor is given by

$$g = \frac{\omega'^2 - \omega''^2}{\omega'^2 + \omega''^2} = \frac{1}{2} \left\{ \frac{\omega_r^2}{\omega'^2} - \frac{\omega_r^2}{\omega''^2} \right\}, \quad (23)$$

where ω' and ω'' are the frequencies at the ends of diameter A'B' perpendicular to OR.

The method of forces in quadrature can be used by adding an eccentric mass m'_0 at a radius r' shifted by $\pi/2$ with respect to the radius r (Fig. 12 a). If $\lambda = (m'_0 r') / (m_0 r)$, the vector diagram plotted for $\lambda = 0$ is also a circle (Fig. 12 b) that crosses the primary circle ($\lambda = 0$) at the point R of amplitude resonance. The damping factor is given by

$$g = \frac{1}{\lambda} \left\{ 1 - \frac{\omega_r^2}{\omega^{*2}} \right\}, \quad (24)$$

where ω_R and ω^* are the frequencies of the point R on the two circles. Crossing the primary circle with two circles plotted using two masses located at $\pm 90^\circ$ permits more accurate location of resonance. The formula for evaluating damping is similar to equation (20).

Ungrounded Systems

Frequency response of ungrounded systems has been analyzed in papers concerned with the measurement of dynamic properties of anti-vibration mountings and road vehicle suspensions. The single-degree-of-freedom base-excited system has been studied (70, 83). In this case the ratio of the tip-to-base amplitude to the base amplitude is expressed by a function similar to equation (21). A two-degree-of-freedom system has been described (13).

The mass-damped spring-mass system (Fig. 13 a) has been analyzed (79). The same system with viscous damping has also been considered (78). Resonance is at the frequency $\omega_r = \left\{ k \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \right\}^{1/2}$, where a minimum of force produces a maximum of relative motion between the two masses. This occurs at point R on the diagram, $\frac{X_1 - X_2}{F_0/k}$, which is a circle. On the diagrams of X_1 or X_2 the resonance frequency corresponds to the points at which the quadrature component is a maximum (Fig. 13 b). This criterion could also be used for systems with many degrees of freedom (79). The damping factor is

$$g = \frac{\omega_2^2 - \omega_1^2}{2\omega_r^2} = \frac{\omega_2^2 - \omega_1^2}{\omega_2^2 + \omega_1^2}, \quad (25)$$

in which ω_1 and ω_2 are the frequencies at the ends of diameter AB perpendicular to OR. It should be noted that, for base-excited systems the method of forces in quadrature combined with the vector diagrams of the direct (X_1/F_0) and transfer (X_2/F_0) receptance locates the points of maximum response amplitude; these do not correspond to the resonance frequency defined above.

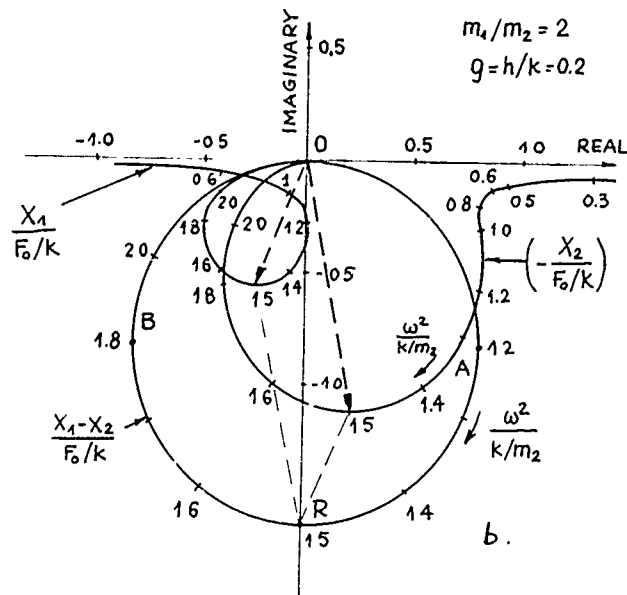
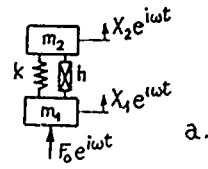


Fig 13

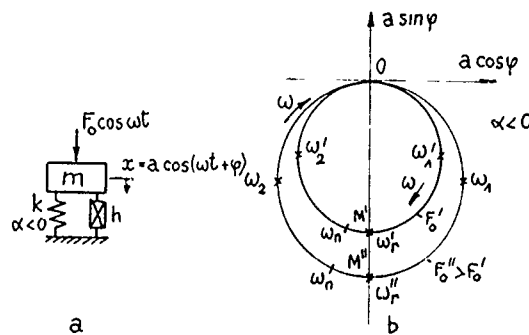


Fig 14

Nonlinear Systems

There are examples in the early literature of studies of the effect of nonlinearities in structural testing (31, 57). De Vries (101, 103-105) plotted the distortion of vector diagrams plotted during tests at relatively large response amplitudes. This procedure was later examined in greater detail by White (113, 115), who showed the limits of the maximum frequency spacing criterion (48) when applied to systems exhibiting hardening elastic characteristics. Billet (6) described the "jump" phenomenon using vector diagrams. A systematic research of the subject was carried out by Hanna and Tobias (38), who showed that the harmonic response of a system with linear damping and nonlinear stiffness should be described by a family of polar diagrams having the excitation force amplitude as a parameter. The points of constant frequency are connected by straight lines only for linear systems. For nonlinear systems, the curvature of the constant-frequency lines is the best indication of whether or not the elastic characteristic is softening or hardening.

The governing equation of motion for method (80) for parameter identification of a system with linear hysteretic damping and cubic stiffness (Fig. 14 a) is

$$m\ddot{x} + \frac{h}{\omega} \dot{x} + k(x + \alpha x^3) = F_0 \cos \omega t. \quad (26)$$

An approximate solution of the form $x = a \cos(\omega t + \phi)$ is sought using the method of harmonic linearization. The vector diagrams of the displacement are circles of the same shape as in $\alpha = 0$ but have a different distribution of the frequency parameter.

On two vector diagrams plotted for different excitation forces F'_0 and $F''_0 > F'_0$ (Fig. 14 b), the frequencies ω'_r and ω''_r of the points of maximum displacement amplitude are determined by combining excitation with forces in quadrature. The best circles are fitted through the plotted points to measure the diameters $a'_r = OM'$ and $a''_r = OM''$. The equivalent hysteretic damping factor is

$$g = \frac{h}{k} = \frac{\omega_2^2 - \omega_1^2}{2} \frac{1 - \left(\frac{a'_r}{a''_r}\right)^2}{\omega_2^2 - \left(\frac{a'_r}{a''_r}\right)^2 \omega_1^2} \quad (29)$$

in which ω_1 and ω_2 are the frequencies of the points corresponding to phase angles $\phi_1 = -45^\circ$ and $\phi_2 = -135^\circ$.

Conclusions

Though vibration testing methods are now directed toward combining experimental testing with computation on digital computers, graphical analysis of vector diagrams using a single exciter have attracted interest and are still extensively used in engineering.

Commercially available instrumentation systems determine both in-phase measurements and quadrature components of the response. The resulting vector diagram is the most widely used form of presentation for the frequency-response functions because of the accuracy with which natural frequencies and damping ratios can be measured. Resonance location criteria and formulas for the evaluation of the damping factor independent of the coordinate axes of the diagram should be used. In general, the best criteria are based on the differentiation of a vector quantity with respect to the frequency.

Reliable techniques have been developed for systems with natural frequencies close to each other or small nonlinearities. The restrictive assumptions concerning damping (magnitude and cross-couplings) are realistic for many systems encountered in practice. Cost and testing time are decreased by methods that use transient excitation and computer aided data-reduction techniques.

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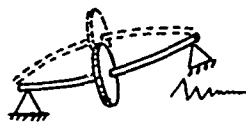
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VIBRATION OF ROTORS THROUGH CRITICAL SPEEDS

TAKUZO IWATSUBA*



The study of the transition of a rotating shaft through a critical speed is essential for the design of high speed rotors, because the maximum amplitude and stress depend on the time required for the rotor to pass through the critical speed. It is thus important to investigate the behavior of the rotor during acceleration and deceleration through critical speeds and to evaluate the maximum amplitude and stress of the shaft.

Sommerfeld(1) was the first to investigate this problem. Experimentally he found a jump phenomenon and instability in transition of the rotor through the critical speed. Since Sommerfeld's work, this problem has been considered by many authors (2, 3, 11, 17, 34). Kononenko systematically studied the interaction of a linear oscillating system with a non-linear oscillating system having a limited energy source and applied the asymptotic method (49) to rotating shafts.

The vibration of a rotor through a critical speed depends on the speed at which the rotors pass through the critical speed. If the driving torque of the rotor is large, the interaction between energy of vibration and energy of rotation is small, and the rotor passes through the critical speed with almost constant acceleration or constant deceleration; in addition, the maximum vibration amplitudes are not large. But, if the driving torque is small--that is, if the driving power is small during acceleration or the dissipating energy is small during deceleration--the energy interaction becomes important. A special case occurs when the interaction of the oscillating system with the energy source is significant and the dissipation and

absorption of the oscillating energy and the rotating energy alter each other. For this reason, the transition through the critical speed of a rotating shaft is classified as one of the following: (i) the vibration of the rotor running through a critical speed with constant acceleration; (ii) the transition (vibration and rotating speed) of a rotor having a limited power supply through a critical speed.

This review deals with the above. Because the transition of a rotor through a critical speed is qualitatively similar to other vibration phenomena, general transition phenomena will be reviewed.

Vibration of a Rotor Running Through a Critical Speed With Constant Acceleration or Deceleration.

Unless a rotor is subjected to large displacements, the effect of transverse vibration on rotational motion is negligible. The angular velocity of a rotor, therefore, changes with constant acceleration or deceleration, and the angle of rotation ϕ is assumed to be

$$\phi = \omega t \pm \delta t^2 \quad (1)$$

where ω is the initial angular velocity of the rotor, and δ the constant coefficient. The angular velocity is expressed

$$\dot{\phi} = \omega \pm 2\delta t \quad (2)$$

A rotor having a single mass (disc) concentrated at the middle and a massless shaft is selected here. Denote the mass of the disc and the spring constant of the shaft as m and k , respectively. The external damping forces are assumed to be proportional to the velocity of the center of the disc, where the damping coefficient is denoted c_e

A fixed coordinate system is chosen with the axes O-xy, such that the x axis is horizontal, and y axis is directed vertically upward. The

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coordinates of the geometric center S are (x_S, y_S) , and those of the center of gravity G at a distance ρ from S are (x, y) . These coordinates are illustrated in Figure 1. The equations of motion of the system are as follows:

$$\begin{aligned} M\ddot{x} + C\dot{x} + kx &= kp \cos(\omega t \pm \delta t^2) \\ M\ddot{y} + C\dot{y} + ky &= kp \sin(\omega t \pm \delta t^2) \end{aligned} \quad (3)$$

If $\delta = 0$, these equations become linear autonomous equations and can be solved easily; but, if $\delta \neq 0$, they become non-autonomous and non-stationary equations, which are very difficult to solve theoretically.

The first attempt to obtain a theoretical solution of the above problem was undertaken (2). method of graphical integration showed that the critical speed is raised and the resonant amplitude reduced by a non-zero rate of acceleration. Both resonant amplitude and critical speed are reduced during the decelerating process. Many papers have been published on theoretical methods (3, 5, 6, 10-16, 20, 28, 37, 40, 42), on experimental methods (7, 36, 44), and on simulation methods (4, 8, 9, 31, 33).

Among the theoretical works, V. A. Grobov (11, 12) and Ellington and McCallion (13) obtained a solution involving Fresnel's integrals. V. A. Grobov (14) later studied a damped system, represented in equation (3). He expressed the displacements as follows:

$$\begin{aligned} x &= \rho \omega_0 \int_0^t e^{-\frac{C}{m}(t-\tau)} \sin \omega_0(t-\tau) \cos(\omega\tau + \rho\tau^2) d\tau + \frac{P}{1 - \frac{\omega^2}{\omega_0^2}} \cos \omega_0 t \\ y &= \rho \omega_0 \int_0^t e^{-\frac{C}{m}(t-\tau)} \sin \omega_0(t-\tau) \sin(\omega\tau + \rho\tau^2) d\tau + \frac{P}{1 - \frac{\omega^2}{\omega_0^2}} \sin \omega_0 t \end{aligned} \quad (4)$$

$$\text{where } \omega_0^2 = \frac{k}{m}.$$

He obtained a solution in closed form using Fresnel's integrals. The analysis was developed numerically by Yanabe and Tamura (40, 42); they divided the process of transition into three periods, namely, precritical speed, near-critical speed, and post-critical speed.

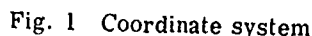
Contrary to Lewis direct graphical integration, Hother-Lushington and Johnson (16) presented a new approach to the problem. They considered the exciting force to be a linear sinusoidal function, the period of which decreased by a small amount after each successive cycle during acceleration. In this method the amplitude and phase of vibration at any time are determined by obtaining the amplitude and phase of vibration produced by each cycle of the exciting force; the several vectors representing these quantities are then added together graphically.

Shimoyama and Yamamoto (7) presented the thing that the critical speed of a rotor system can be obtained from a beat oscillation that occurs throughout the critical speed; they also stated that amount of eccentricity and the phase angle of eccentricity can be obtained from the beat oscillation.

With the simulation technique, Baker (4) used the differential analyzer at the Massachusetts Institute of Technology. He stressed the importance of proper selection of a spring constant, damping coefficient, and mass of suspended unit as a means for avoiding serious vibration while bringing a heavily unbalanced spring-mounted rotor up to speed. McCann et al (8, 9), Quazi and MacFarlane (31), and Hirano and Matsukura (33) used electric analog computers on rotor problems. Quazi and MacFarlane studied the effect of driving torque, the comparison of acceleration with deceleration, the effect of eccentricity and damping, and the effect of the degree of acceleration; they also showed a feedback control system device, which keeps the shaft deflection within stipulated limits.

The Vibration of a Rotor Running Through a Critical Speed With a Limited Power Supply.

Some years after the existence of the jump phenomenon and the instability of a rotor running through the critical speed were first found, Timoshenko repeated Sommerfeld's experiments



The vibration of a rotor driven by a non-ideal energy source with a known speed characteristic is now considered. The rotor has a weightless shaft with a lumped spring constant and a rigid disc with a lumped mass and a moment of inertia. The system without the rotor disc also has a moment of inertia. The external damping forces are assumed to be proportional to the velocity of the center of the disc; the internal damping forces are assumed to be proportional to the velocity of the bending deformation of the shaft. Driving torque is assumed to be a function of the rotational speed.

$$M\ddot{\chi} + \kappa\dot{\chi} = m\rho\dot{\phi}^2 \cos\phi - (C_e + C_i)\dot{\chi} - C_i\dot{\phi}y$$

$$I\ddot{\phi} = L(\dot{\phi}) - q(\dot{\phi}) - Ci(\dot{\chi}y - \dot{\chi}y) - Ci\dot{\phi}(\dot{\chi}^2 + y^2) - k\rho(\chi \sin\phi - Y \cos\phi)$$

$$\begin{aligned}\ddot{X} + X &= \dot{\phi}^2 \cos \phi - 2(D_1 + D_2) \dot{X} - 2D_2 \dot{\phi} Y \\ \ddot{Y} + Y &= \dot{\phi}^2 \sin \phi - 2(D_1 + D_2) \dot{Y} - 2D_2 \dot{\phi} X - G \\ \ddot{\phi} &= \delta^2 \{ R - S\dot{\phi} - T\dot{\phi}' - 2D_2(\dot{X}Y - Y\dot{X}) - \\ &\quad - 2D_2\dot{\phi}(X' + Y') - (X \sin \phi - Y \cos \phi) \} \end{aligned}$$

(6)

$$\delta = \rho/j \quad (j = \sqrt{I/m}),$$

$$\eta \approx \phi/\omega = \dot{\phi}$$

$$D_1 \approx C_p / 2 \pi \omega,$$

$$D_z = C_i / 2 \pi \omega,$$

$$R-S\dot{\phi} = L(\dot{\phi}) / k\rho^2,$$

$$T\dot{\phi}^2 = q(\dot{\phi}) / k\rho^2,$$

$$\tau = \omega t,$$

X, Y = Non-dimensional coordinate.

G = Non-dimensional gravity acceleration.

Using the complex variables $Z = X + iY$,
 $\bar{Z} = X - iY$, equation (6) are
transformed to

$$\ddot{Z} + Z = \dot{\phi}^2 e^{i\phi} - 2(D_1 + D_2) \dot{Z} + 2iD_2 \dot{\phi} Z - iG$$

$$\ddot{\phi} = \delta^2 \{ R - S\dot{\phi} - T\dot{\phi}^2 + iD_2 (Z\ddot{Z} - \dot{Z}\dot{\bar{Z}}) - 2D_2 \dot{\phi} Z\bar{Z} - \frac{1}{2} \{ (Z + \bar{Z}) \sin\phi + i(Z - \bar{Z}) \cos\phi \} \}$$

(7)

Since the vibration of rotors is important in the neighborhood of the main critical speed, this region will be considered in the solution of equations (7). The gravitational force is considered negligible since it has no significant effect on the whirling of the rotors in this region of shaft speed.

Assuming the right-hand sides of equations (7) are small--i.e., the changes of amplitude, phase lag, and rotational frequency are small during the period of rotation of the shaft--the asymptotic method of Bogoliubov and Mitropolskii can be used.

The solution of the equation is expressed in the form

$$Z = A \cos\phi + B \sin\phi,$$

$$\dot{Z} = -A \sin\phi + B \cos\phi,$$

$$\bar{Z} = \bar{A} \cos\phi + \bar{B} \sin\phi$$

$$\dot{\bar{Z}} = -\bar{A} \sin\phi + \bar{B} \cos\phi$$

(8)

where A and B are complex quantities, and \bar{A} and \bar{B} are their conjugates. In these equations it was assumed that

The derivative of the first expression of equations (8) can be written:

$$\dot{Z} = \frac{dA}{d\tau} \cos\phi - \dot{\phi} A \sin\phi + \frac{dB}{d\tau} \sin\phi + \dot{\phi} B \cos\phi$$

(9)

After transformation of equations (7) the following are obtained.

$$\frac{dA}{d\tau} = (-\dot{\phi}) B - (\dot{\phi}^2 e^{i\phi} + 2(D_1 + D_2) (A \sin\phi - B \cos\phi) + 2iD_2 \dot{\phi} (A \cos\phi + B \sin\phi))$$

$$\frac{dB}{d\tau} = - (-\dot{\phi}) A - (\dot{\phi}^2 e^{i\phi} + 2(D_1 + D_2) (A \sin\phi - B \cos\phi) + 2iD_2 \dot{\phi} (A \cos\phi + B \sin\phi))$$

$$\begin{aligned} \frac{d\phi}{d\tau} &= \delta^2 \{ R - S\dot{\phi} - T\dot{\phi}^2 + iD_2 (A\bar{B} - \bar{A}B) \\ &+ 2iD_2 \dot{\phi} \{ A\bar{A} \cos^2\phi + B\bar{B} \sin^2\phi + (\bar{A}B + A\bar{B}) \sin\phi \cos\phi \} \\ &- \frac{1}{4} \{ i(A - \bar{A}) + (B + \bar{B}) + i(A + iB) e^{-4i\phi} \\ &- i(\bar{A} - i\bar{B}) e^{4i\phi} \} \} \end{aligned}$$

(10)

Using the perturbation method, an approximate solution of equations (10) can be obtained in the form

$$A = A_1 + \epsilon U_1 (A_1, B_1, \bar{A}_1, \bar{B}_1, \eta)$$

$$B = B_1 + \epsilon U_2 (A_1, B_1, \bar{A}_1, \bar{B}_1, \eta)$$

$$\phi = \eta + \epsilon U_3 (A_1, B_1, \bar{A}_1, \bar{B}_1, \eta).$$

(11)

The principal parts of this solution--the quantities A_1 , B_1 , and η --are determined from the first approximate equations. These are obtained by averaging the right-hand side of equations (10) for ϕ , where ϕ is assumed to be constant over the period of oscillation. As a result of the above procedure, the following equations are obtained.

$$\frac{dA_1}{dt} = (1 - \eta) B_1 - \{ (D_1 + D_2) A_1 + i D_2 \eta B_1 + \frac{1}{2} \eta^2 \}$$

$$\frac{dB_1}{dt} = - (1 - \eta) A_1 - \{ (D_1 + D_2) B_1 - i D_2 \eta A_1 - \frac{1}{2} \eta^2 \}$$

$$\frac{d\eta}{dt} = \delta^2 (f_R - S\eta - T\eta^2 + D_2 (A_1 \bar{B}_1 - \bar{A}_1 B_1) - D_2 \eta (A_1 \bar{A}_1 + B_1 \bar{B}_1) - \frac{1}{2} \{ i (A_1 - \bar{A}_1) + (B_1 + \bar{B}_1) \}^2).$$

(12)

Equations (7) could be rewritten by using the asymptotic method in equations (12) in the neighborhood of the critical speed. These approximate equations are used in later analyses.

The expressions for amplitude, phase lag, and rotational frequency in the steady state are obtained by setting the derivatives of equation

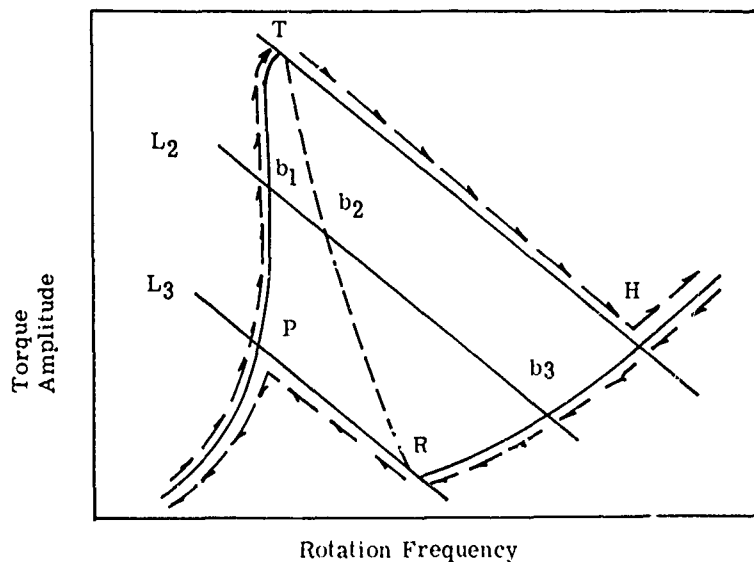


Fig. 2 Torque relation for linear system

(12) equal to zero.

The following equations are introduced:

$$A_1 + A_r - iA_1' \quad A_1 = A_r + iA_1'$$

$$B_1 = B_r + iB_1' \quad B_1 = B_r - iB_1'$$

Thus X and Y are expressed as

$$X = a \cos (\phi + \theta) \quad Y = a \sin (\phi + \theta) \quad (13)$$

and α and θ are

$$\alpha = \frac{\eta^2 \sqrt{\{(1-\eta)^2 + 4\{D_1 + D_2(1-\eta)\}^2\}}}{(1-\eta^2)^2 + 4\{D_1 + D_2(1-\eta)\}^2} \quad (14)$$

The following equation for rotation frequency can be obtained from equations (12).

$$R - S\eta - T\eta^2 + 2D_2(1-\eta)(A_r^2 + A_i^2) + A_i = 0. \quad (15)$$

Since the expressions A_r and A_i include the variable η , equation (15) cannot be solved explicitly for η . But η can be determined from the relationship between the driving torque $R - S\eta$, the friction resisting torque $-T\eta^2$, and the resisting torque due to the vibration. Equation (15) can be rewritten:

$$L(\phi) - q(\phi) - C_e \omega^2 = 0, \quad (16)$$

where $L(\phi)$ represents the driving torque of the rotor, $q(\phi)$ is the resisting torque, and C_e is the coefficient of external linear damping.

In the analysis of non-stationary motion, a perturbation method is used for equations (12). From the relationship $B_1 = iA_1$, equations (12) can be reduced to a system of real equations. The perturbed motion can also be written as

$$\eta = \eta_s + \eta_1$$

$$A_r = A_{rs} + A_{r1}$$

$$A_i = A_{is} + A_{i1}$$

where η_s, A_{rs}, A_{is} are values of η, A_r, A_i for stationary motion; and η_1, A_{r1}, A_{i1} are small perturbations of the quantities η, A_r, A_i from their values for stationary motion. The equation of perturbation terms is

$$\frac{d}{dt} \begin{Bmatrix} \eta \\ A_{r1} \\ A_{i1} \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{Bmatrix} \eta_1 \\ A_{r1} \\ A_{i1} \end{Bmatrix} \quad (18)$$

where b_{ij} are expressed by $S, T, \eta, D_1, D_2, A_r, A_i$, and D . Equation (18), the characteristic equation of the system, is

$$\lambda^3 + B_1 \lambda^2 + B_2 \lambda + B_3 = 0, \quad (19)$$

where B_1, B_2, B_3 are constants expressed by b_{ij} ($i, j = 1, 2, 3$). The necessary and sufficient condition for the stationary motion to be stable is

$$B_1 > 0, B_1 B_2 - B_3 > 0, B_3 > 0 \quad (20)$$

From expression (20) stability criteria can be made.

Kononenko (49) gave a clear qualitative explanation of this type of instability for various vibrating systems with a non-ideal energy source. This explanation is summarized as follows. The condition of stability is written

$$\frac{d}{d\eta} \{ L(\eta) - S(\eta) \} < 0 \quad (21)$$

where $L(\eta)$ is the driving torque, and $S(\eta)$ is the resisting torque. This is identical with the condition to be satisfied between a driving torque and a load for the stable, steady operation of any rotating machine.

In Figure 2, for example, the points b_1 and b_3 are stable, but b_2 is not stable in this respect.

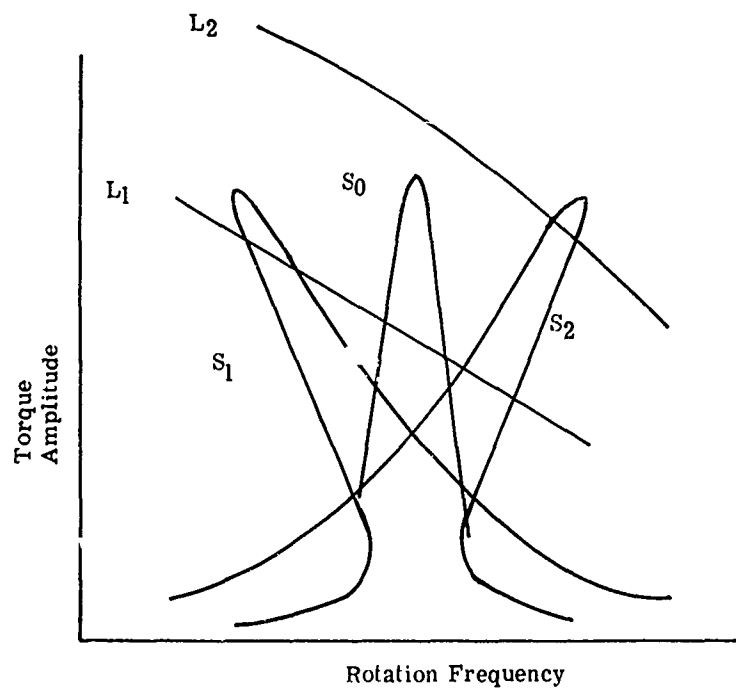


Fig. 3 Torque relation for nonlinear system

If L_1 and L_3 are the boundary characteristics and if the system is in a state characterized by the unstable point T, it will jump from T to H which is the only stable point having the same characteristic L_1 . At point T the system undergoes non-stationary motion. Similarly, at point R a non-stationary transition to point P takes place since P is the only stable point having the characteristic L_3 . (These jumps are similar to those which take place in the forced vibration of an ideal nonlinear system.) These jumps are the reason why in many practical cases the realization of part of the resonance curve is not possible.

Subsequent refinements of this problem have been obtained from a study of nonlinear oscillation systems by Kononenko (22, 23) and Hubner (29); a study of an oscillating system with many degrees of freedom by Kononenko (21); studies of an oscillating system having a rotor with unequal shaft stiffness by Goeskokov (48) and Iwatsubo and by Kanki and Kawai (34, 38, 41); a study of distributed mass systems by Kotera (43); and a study of the design of a rotor system by Matsuura (35, 45). The problem of the interaction between a nonlinear oscillating system and an energy source is summarized in Figure 3. It has been shown that the resonance properties depend a great deal on the properties of the energy source.

Iwatsubo et al theoretically and experimentally studied the interacting system by using the asymptotic method developed by Kononenko. They concluded that for the rotor with asymmetric shaft stiffness: (i) an asymmetric rotor system with a limited power supply has an unstable region not only due to shaft asymmetry (i.e., unstable region of free vibration) but also due to the characteristics of the energy source--both unstable regions increase according to the increase of shaft asymmetry, but the phase angle of eccentricity affects only the latter unstable region. (ii) the non-stationary vibration of an asymmetric rotor is much affected by the phase angle of eccentricity. (iii) when the capacity of the energy source is small, the interaction between rotor and energy source increases as shaft asymmetry increases so that a decrease of rotational frequency during passage through a critical speed may occur. (iv) maximum amplitude of non-stationary vibration during acceleration of a rotor with a limited power supply through a critical speed is greater than with constant

acceleration with the corresponding torque. (v) the shaft asymmetry has a reverse effect against damping for maximum amplitude of non-stationary vibration of a rotor through the critical speed.

In summary, working guidelines have been given for a rotating shaft passing through a critical speed. The interaction between an oscillating system and an energy source cannot be avoided. Therefore, suitable controls must be made in this system. Control variables while rotor is passing through the critical speed will be the driving torque, damping force and shaft stiffness.

The interaction between an energy source and a rotor system having journal bearings is also an important problem because the bearings dissipate much energy in proportion to a rotating speed and have hard non-linear properties as a function of rotating speed.

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BOOK REVIEWS

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REDUCTION OF MACHINERY NOISE

Edited by M. J. Crocker

Purdue Univ., West Lafayette, Inc. (1974)

This volume contains the proceedings of two short courses held at Purdue University in 1974 on "Fundamentals of Noise Control" and "Reduction of Machinery Noise." The book contains 31 papers and is divided into three sections: Fundamentals of Noise Control, Reduction of Machinery Noise, and Applied Noise Case Histories.

Part I, Fundamentals of Noise Control, begins with pertinent definitions and descriptions of instrument and noise measurements, the physiology of hearing, and ways by which the ears can be damaged. It continues with a discussion of noise legislature, possible approaches to noise reduction and the proper application of vibration isolation, and the use of enclosures and barriers to absorb sound. The various concepts are adequately explained and the design engineer should have no difficulty in interpreting them. In the reviewer's opinion more information should have been given about isolators; in addition, jet noise reduction should have been considered since it is one of the most objectionable of public noise sources.

Part II delves into some important aspects of machinery noise reduction. The correct interpretation of noise data obtained from instruments is stressed because it influences structural design and methods used to attenuate undesirable noise. Part II contains an excellent discussion of truck noise; gearbox noise and diesel engine noise are considered, but tire noise is not discussed. The finite element method is used to model the engine block for calculations of frequencies and mode shapes. These results are applied directly in the redesign of a section of the engine block: a healthy approach that mitigates the "brute force testing" concept.

Part II also contains discussions of methods for reducing machinery noise that do not adversely affect the machine or its performance. Possible solutions to reducing machine shop noise, especially that associated with punch presses, are considered. This section concludes with discussions of reduction of centrifugal machinery noise and fan and blower noise, control of the noise of large steam turbine generators, and aspects of aerodynamic noise in control valve design. The reviewer heartily recommends this part of the book.

The concluding part of the book is concerned with noise reduction procedures and tests associated with such appliances as chain saws, vacuum cleaners, household refrigerators, air conditioners, and clocks. The reviewer feels that this section is too short. Case histories of lawn mowers, snowblowers, dishwashers, and showmobiles should have been included, as well as means and methods for reducing the noise associated with their operation.

The volume although not an exhaustive treatment, is a good compendium of modern-day noise control and reduction of machinery noise. The information contained in the book is useful without overwhelming the reader. It is not intended to make the reader a specialist but is recommended for, and should be of interest to, designers who must consider noise control.

H. Saunders
General Electric Co.
Large Steam Turbine & Generator
Schenectady, NY 12305

ELEMENTARY ENGINEERING FRACTURE MECHANICS

by David Broek

Noordhoff International
Publishing, Leyden,
The Netherlands

This book will indeed be welcomed by those interested in fracture mechanics. Very few texts are available in this field, and this book will help fill the void that currently exists for formal textbooks on fracture mechanics. The book is suitable for advanced undergraduates or first year graduate students and will also be useful to those outside the classroom as a general introduction to fracture mechanics. The practical application of fracture mechanics, is emphasized; but the subject is treated so that it will appeal to both engineers and metallurgists.

The book is divided into two major parts. Part I treats basic principles and consists of the following chapters:

1. Summary of basic problems and concepts.
2. Mechanisms of fracture and crack growth
3. The elastic crack-tip stress field
4. The crack tip plastic zone
5. The energy principle
6. Dynamics and crack arrest
7. Plane strain fracture toughness
8. Plane stress and transitional behavior
9. The crack opening displacement criterion
10. Fatigue crack propagation
11. Fracture resistance of materials

Part II is an introduction for applying fracture mechanics to real problems and contains the following chapters:

12. Fail-safety and damage tolerance
13. Determination of stress intensity factors
14. Practical problems
15. Fracture of structures
16. Stiffened sheet structures

Each chapter has complete bibliography.

It is important to note that fracture mechanics is still in early stages of development, and significant advances are made continuously. Thus, although this text provides an excellent introduction to anyone investigating the field, constant review of the current literature is necessary for keeping abreast with new technology.

Steven E. Benzley
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CALENDAR			
MEETING	DATE 1978	LOCATION	CONTACT
9th Annual Simulation Symposium, IEEE/Assn for Computing Mech./Soc. for Computer Simulation	<u>MAR.</u> 17-19	Tampa, Fla	I. Kay, Box 22573, Tampa, Fla 33622
Joint Gas Turbine and Fluids Engineering Conference, ASME	21-25	New Orleans, La	M. Churchill, ASME Hq.
Diesel and Gas Engine Power Conference and Exhibit, ASME	<u>APR.</u> 4-8	Chicago, Ill.	ASME Hq
INTER-NOISE 76 [NOISE CONTROL ENGINEERING] (concurrent with Acoustical Soc. of Amer. Spring Meeting), INCE/ASA	5-7	Washington, D. C.	INTER-NOISE 76, c/o ARL PSU, Box 30, State College, Pa 16801
Communications Satellite Systems Conference and Display, AIAA/CASI	5-8	Montreal, Canada	W. Brunke, AIAA Hq.
Joint Railroad Technical Conference, IEEE/ASME	6-8	Chicago, Ill	ASME Hq., G. Jannon, IEEE Hq
Spring Meeting, ASA	6-9	Washington, D. C	Mrs. P. G. Weissler, Natl. Bur. Stds., Washington, D. C. 20234
American Power Conference, Ill. Inst. Tech.	19-21	Chicago, Ill	R. A. Budenholzer, DIR, APC, IIT, 10 W. 35th St., Chicago, Ill. 60616
Environmental Technology '76, 22nd Annual Equipment Exposition and Technical Meeting, IES	25-28	Philadelphia, Pa.	IES Hq.
22nd Technical Meeting and Equipment Exposition, IES	26-29	Philadelphia, Pa	Betty L. Peterson, IES Hq.
Industrial Power Conference (Biennial), ASME	<u>MAY</u> May	Houston, Tex.	ASME Hq
4th National Conference on Composite Materials Testing and Design, ASTM	3-4	Valley Forge, Pa	Ms. J. B. Wheeler, ASTM Hq.
17th Structures, Structural Dynamics and Materials Conference and Display, AIAA/ASME/SAE	5-7	Valley Forge, Pa.	AIAA Hq
Spring Meeting and Exposition, SESA	9-14	Washington, D. C	B. E. Rossi, SESA Hq.
Design Engineering Conference, ASME	10-13	Chicago, Ill	P. Drummond, ASME Hq
Air Transportation Meeting, SAE	11-13	New York, N. Y.	SAE Hq
International Convention (ELECTRO), IEEE	11-14	Boston, Mass.	W. C. Weber, Jr., IEEE Hq.
Aerospace Electronics Conference (NAECON 76), IEEE/AIAA	18-20	Dayton, Ohio	J. E. Singer, NAECON 76, c/o IEEE Dayton Sec Section, 140 E. Monument Ave., Dayton, Ohio 45402
30th Annual Technical Conference, ASQC	24-26	Toronto, Canada	R. W. Shearman, ASQC Hq
Lubrication Symposium, ASME	24-26	Atlanta, Ga.	ASME Hq
Engineering Mechanics Specialty Conference, ASCE	26-28	Waterloo, Canada	ASCE-EM Spec. Conf., c/o Solid Mech. Div., Univ. Waterloo, Waterloo, Canada N2L3G1
Applied Mechanics Conference, ASME	<u>JUNE</u> 14-17	Salt Lake City, Utah	ASME Hq.
1976 Spring Meeting, SNAME	2-4	Philadelphia, Pa.	R. G. Mende, SNAME Hq.
30th Annual Technical Conference, ASQC	7-9	Toronto, Canada	R. W. Shearman, ASQC Hq
Summer Computer Simulation Conference, AEChE/ISA/SHARE/AIAA/Amer. Meteorol. Soc./Soc. for Computer Sim./Amer. Geophys. Union	<u>JULY</u> 11-14	Washington, D. C	A. Rubin, Electnics. Assoc. Inc., 185 Monmouth Pkwy., W. Long Branch, N. J.
6th Intersociety Conference on Environmental Systems, SAE	12-15	San Diego, Calif.	SAE Hq.
Power Engineering Society Summer Meeting, IEEE	18-23	Portland, Ore.	D. A. Gillies, Bonneville Power Admin., Box 491, Vancouver, Wash. 98660
International Symposium on Earthquake Structural Engineering, Univ. of Missouri	<u>AUG.</u> 23-25	St. Louis, Mo.	Dr. F. Y. Cheng, Dept. of CE, Univ. of Missouri, Rolla, Mo.
82nd National Meeting, AIChE	29-1	Atlantic City, N. J.	J. Henry, AIChE Hq.
Petroleum and Chemical Industry Technical Conference, IEEE	30-1	Philadelphia, Pa.	IEEE Hq. (tentative)
Computer Society 1978 Fall Meeting (COMPCON Fall 78), IEEE	<u>SEPT.</u> 8-11	Washington, D. C	H. Haymann, Box 639, Silver Spring, Md. 20901
11th Intersociety Energy Conversion Engineering Conference, IEEE/ASME/AIChE/ANS/SAE/ACS/AIAA	12-17	Stateline, Nev.	J. Henry, AIChE Hq.
Joint Power Generation Technical Conference, IEEE/ASME/ASCE	19-22	Buffalo, N. Y.	R. C. Seebald, Niagara Mohawk Pwr. Co., 535 Washington St., Buffalo, N. Y. 14203
Design Engineering Technical Conference, ASME	26-29	Montreal, Canada	ASME Hq.

CALENDAR			
MEETING	DATE 1976	LOCATION	CONTACT
Joint Lubrication Conference, ASME	<u>OCT.</u> 5-7	Boston, Mass	ASME Hq
20th Annual Stapp Car Crash Conference, SAE	13-15	Detroit, Mich	SAE Hq.
47th Shock and Vibration Symposium	19-21	Albuquerque, N M	SVIC
84th Annual Meeting, SNAME	<u>NOV</u> 11-13	New York, N. Y.	R. G. Mende, SNAME Hq
Fall Meeting, ASA	16-19	San Diego, Calif	ASA Hq
Winter Annual Meeting, ASME	28-3	New York, N. Y.	AMSE Hq
Automotive Engineering Congress and Exposition (SAE Annual Meeting), SAE	<u>1977</u> <u>FEB.</u> 28-4	Detroit, Mich.	SAE Hq
Gas Turbine Conference and Products Show, ASME	<u>MAR.</u> 27-31	Philadelphia, Pa	ASME Hq
Joint Railroad Conference, IEEE/ASME	30-2	Washington, D C	IEEE Hq
American Power Conference, Ill Inst Tech	<u>APR</u> 18-20	Chicago, Ill	R. A. Budenholzer, Dir. APC, c/o IIT, 10 W 35th St., Chicago, Ill 60616
Diesel and Gas Engine Power Conference and Exhibit, ASME	24-28	Dallas, Tex	ASME Hq
31st Annual Technical Conference, ASQC	<u>MAY</u> 16-18	Philadelphia, Pa	R. W. Shearman, ASQC Hq
Lubrication Symposium, ASME	<u>JUNE</u> June	St. Louis, Mo	ASME Hq
Fuels and Lubricants Meeting, SAE	7-9	Tulsa, Okla	SAE Hq.
Applied Mechanics Conference, ASME	14-16	New Haven, Conn	ASME Hq

CALENDAR ACRONYM DEFINITIONS AND ADDRESSES OF SOCIETY HEADQUARTERS

AFIPS	American Federation of Information Processing Societies 210 Summit Ave., Montvale, N J 07645	CCCCAM	Chairman, c/o Dept. ME, Univ. Toronto, Toronto 5, Ontario, Canada
AGMA	American Gear Manufacturers Association 1330 Mass Ave., N. W., Washington, D. C.	IEEE	Institute of Electrical and Electronics Engineers 345 E 47th St., New York, N Y 10017
AIAA	American Institute of Aeronautics and Astronautics 1290 Sixth Ave., New York, N.Y. 10019	IES	Institute Environmental Sciences 940 E Northwest Highway, Mt. Prospect, Ill. 60056
AIChE	American Institute of Chemical Engineers 345 E 47th St., New York, N.Y. 10017	IFTOMM	International Federation for Theory of Machines and Mechanisms US Council for TMM, c/o Univ. Mass., Dept. ME, Amherst, Mass 01002
AREA	American Railway Engineering Association 59 E Van Buren St., Chicago, Ill 60605	INCE	Institute of Noise Control Engineering P O Box 3206, Arlington Branch, Poughkeepsie, N.Y. 12603
AHS	American Helicopter Society 30 E. 42nd St., New York, N Y 10017	ISA	Instrument Society of America 400 Stanwix St., Pittsburgh, Pa. 15222
ARPA	Advanced Research Projects Agency	ONR	Office of Naval Research Code 400B4, Dept. Navy, Arlington, Va. 22217
ASA	Acoustical Society of America 335 E. 45th St., New York, N.Y. 10017	SAE	Society of Automotive Engineers 400 Commonwealth Drive, Warrendale, Pa. 15096
ASCE	American Society of Civil Engineers 345 E 45th St., New York, N.Y. 10017	SEE	Society of Environmental Engineers 6 Conduit St., London W1R 9TG, England
ASME	American Society of Mechanical Engineers 345 E. 47th St., New York, N.Y. 10017	SESA	Society for Experimental Stress Analysis 21 Bridge Sq., Westport Conn 06880
ASNT	American Society for Nondestructive Testing 914 Chicago Ave., Evanston, Ill 60202	SNAME	Society of Naval Architects and Marine Engineers 74 Trinity Pl., New York, N Y 10006
ASQC	American Society for Quality Control 161 W. Wisconsin Ave., Milwaukee, Wis. 53203	SVIC	Shock and Vibration Information Center Naval Research Lab Code 8404, Washington, D C 20375
ASTM	American Society for Testing and Materials 1916 Race St., Philadelphia, Pa 19103	URSI-USNC	International Union of Radio Science - US National Committee c/o MIT Lincoln Lab., Lexington, Mass 02173